**3**. [12 points] Consider the family of linear functions

$$L(x) = ax - 3$$

and the family of functions

$$M(x) = a\sqrt{x}$$

where $a$ is a nonzero constant number. Note that the number $a$ is the same for both equations. Find a value of $a$ for which $L(x)$ is tangent to the graph of $M(x)$. Also find the $x$ and $y$ coordinates of the point of tangency. Write your answers in the blanks provided.

Solution: Since the graphs of $M$ and $L$ intersect are tangent at the relevant point, we have

$$M'(x) = L'(x). \qquad (\dagger)$$

Computing the derivatives, we get

$$\frac{1}{2}ax^{-\frac{1}{2}} = a$$

Canceling the $a$ from this equation and doing some algebra, we see $x = \frac{1}{4}$. Since the two graphs must also intersect at this point, we must have

$$M\left(\frac{1}{4}\right) = L\left(\frac{1}{4}\right),$$

and using this, we can solve for $a$; we get $a = -12$. Finally, we can recover $y = -6$ by plugging these values into either the equation for $M(x)$ or the equation for $L(x)$.

$a =$ **-12**

$x =$ **.25**

$y =$ **-6**