

4. [11 points]

a. [4 points] Find the tangent line approximation of the function

$$p(x) = 1 + x^k$$

near $x = 1$, where k is a positive constant.

Solution:

$$L(x) = k(x - 1) + 2$$

b. [2 points] Suppose you want to use your tangent line from (a) to approximate the number $1 + \sqrt{0.95}$. What values of k and x would you plug in to your answer from (a)?

Solution: We'd take $k = \frac{1}{2}$ and $x = .95$.

c. [2 points] Approximate $1 + \sqrt{0.95}$ using your tangent line from (a).

Solution: We have

$$1 + \sqrt{.95} \approx .5(-.05) + 2 = 1.975.$$

d. [3 points] Determine whether your approximation in (c) is an over- or underestimate. Be sure your reasoning is clear.

Solution: The graph of $1 + x^{.5}$ is just the graph of the square root function shifted up by one, so it's concave down everywhere. It follows that the linear approximation is an overestimate.