**7.** [10 points] For each real number $k$, there is a curve in the plane given by the equation

$$e^{y^2} = x^3 + k.$$

**a.** [4 points] Find $\frac{dy}{dx}$.

Solution: We have

$$2ye^{y^2}\frac{dy}{dx} = 3x^2,$$

so

$$\frac{dy}{dx} = \frac{3x^2}{2ye^{y^2}}$$

**b.** [3 points] Suppose that $k = 9$. There are two points on the curve where the tangent line is horizontal. Find the $x$ and $y$ coordinates of each one.

Solution: Horizontal tangent lines occur when the numerator of the derivative is zero, so in this case $x = 0$. To solve for the $y$-coordinate, we have

$$e^{y^2} = 9$$

so $y = \pm\sqrt{\ln(9)}$.

**c.** [3 points] Now suppose that $k = \frac{1}{2}$. How many points are there where the curve has a horizontal tangent line?

Solution: Again we get $x = 0$. Now if we try to solve for $y$ we have

$$y^2 = \ln\left(\frac{1}{2}\right) < 0$$

and so there are no points where the curve has a horizontal tangent line.