

11. [5 points] A curve  $C$  gives  $y$  as an implicit function of  $x$ . The curve  $C$  passes through the point  $(1, 2)$  and satisfies

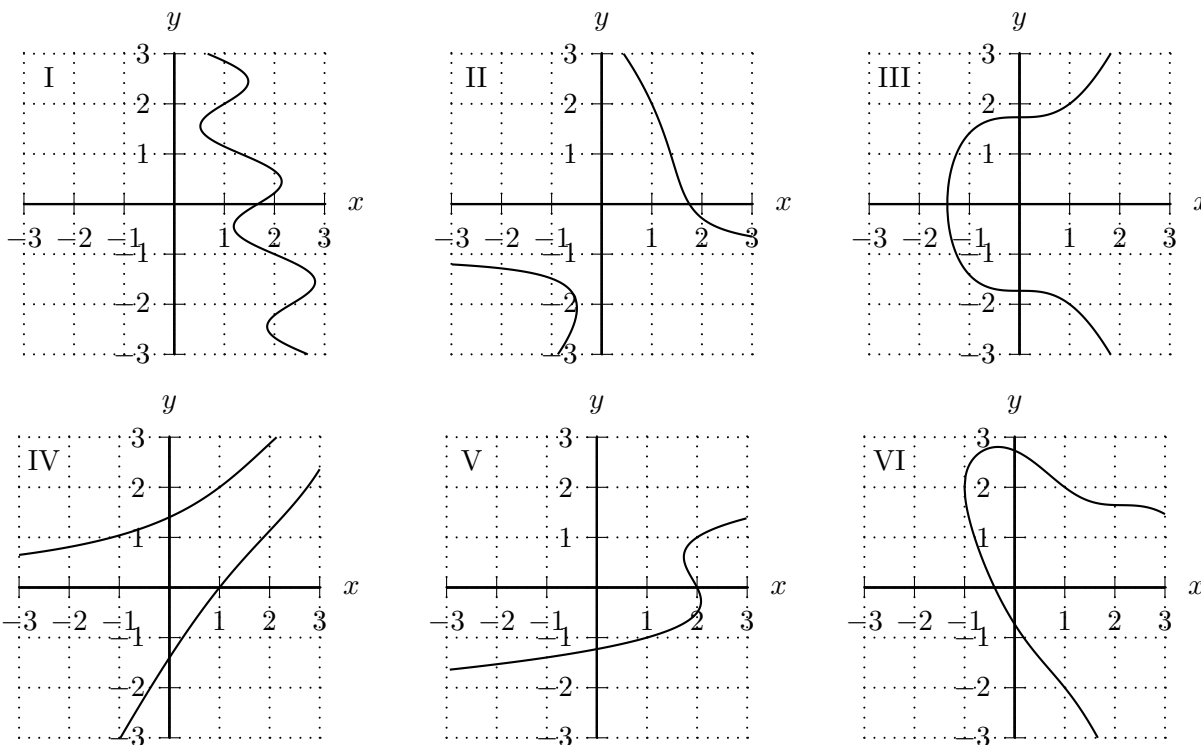
$$\frac{dy}{dx} = \frac{y^2 - 2xy + 4y - 5}{4(y - x)}.$$

- a. [1 point] One of the values below is the slope of the curve  $C$  at the point  $(1, 2)$ . Circle that one value.

*Solution:* Plugging  $x = 1$  and  $y = 2$  into the given formula for  $\frac{dy}{dx}$  yields  $3/4$ .

**Answer:** The slope at  $(1, 2)$  is  $\frac{1}{4}$     $\frac{1}{3}$     $\frac{1}{2}$     $\frac{5}{8}$     $\frac{2}{3}$     $\frac{3}{4}$     $\frac{4}{5}$

- b. [4 points] One of the following graphs is the graph of the curve  $C$ . Which of the graphs I-VI is it? To receive any credit on this question, you must circle your answer next to the word “Answer” below.



*Solution:* We know that the desired curve passes through the point  $(1, 2)$  with slope  $3/4$ . This allows us to eliminate Graph V (which doesn't pass through  $(1, 2)$ ) and Graphs II and VI (which have negative slope at  $(1, 2)$ ).

To decide between Graphs I, III, and IV, we look at other points on the graphs.

Graph I passes through the point  $(2, -1)$  with negative slope, but the above formula for  $\frac{dy}{dx}$  says that it should have positive slope there, so Graph I is incorrect.

Graph III passes through the point  $(1, -2)$  with negative slope, but the above formula for  $\frac{dy}{dx}$  says that it too should have positive slope there, so Graph III is incorrect.

The only remaining possibility is Graph IV.

(Note that we could have also eliminated all but Graph IV by checking for vertical tangent lines at points  $(x, y)$  with  $y = x$ .)

Remember: To receive any credit on this question, you must circle your answer next to the word “Answer” below.

**Answer:**   I     II     III   IV   V   VI