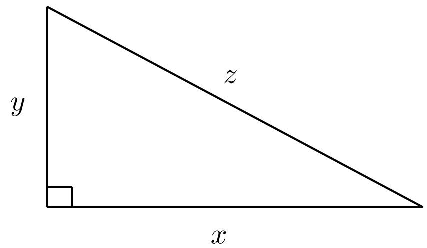
2. [9 points] Consider a right triangle with legs of length x ft and y ft and hypotenuse of length z ft, as in the following picture:



a. [2 points] Suppose that the perimeter of the triangle is 8 ft. Let $A(x)$ give the area of the triangle, in ft^2, as a function of the side length x. In the context of this problem, what is the domain of $A(x)$? *Note that you do not need to find a formula for $A(x)$.*

Solution: Notice that we can let x be arbitrarily close to 0 and still have a perimeter of 8 ft by making y and z both very close to 4.
However, since z is always at least as big as x and since y is positive, x cannot be larger than 4 or or else $x + y + z$ would be greater than 8 ft.

Answer: $(0, 4)$

b. [7 points] Suppose instead that the perimeter of the triangle is allowed to vary, but the area of the triangle is fixed at 3 ft^2. Let $P(x)$ give the perimeter of the triangle, in ft, as a function of the side length x.

(i) In the context of this problem, what is the domain of $P(x)$?

Solution: x must be positive, but there is no upper bound on x. Even if x is very large, with a small enough y, it is still possible for the triangle to have area 3 ft^2.

Answer: $(0, \infty)$

(ii) Find a formula for $P(x)$. The variables y and z should not appear in your answer. (This is the equation one would use to find the value(s) of x minimizing the perimeter. You should not do the optimization in this case.)

Solution: The perimeter of the triangle is $x+y+z$ ft. Since we want it to be a function of x only, we need to use other information to eliminate the other variables.

The area is 3 ft^2, so we have $\frac{1}{2}xy = 3$. Solving for y yields $y = \frac{6}{x}$.

Since this is a right triangle, by the Pythagorean Theorem, we have $x^2 + y^2 = z^2$. Solving for z yields $z = \sqrt{x^2 + y^2}$. Using $y = \frac{6}{x}$ allows us to write z in terms of x as $z = \sqrt{x^2 + \left(\frac{6}{x}\right)^2}$.

Finally, then, we have

$$P(x) = x + y + z = x + \frac{6}{x} + \sqrt{x^2 + \left(\frac{6}{x}\right)^2}.$$

Answer: $P(x) = x + \dfrac{6}{x} + \sqrt{x^2 + \left(\dfrac{6}{x}\right)^2}$