4. [8 points] A ship’s captain is standing on the deck while sailing through stormy seas. The rough waters toss the ship about, causing it to rise and fall in a sinusoidal pattern. Suppose that \( t \) seconds into the storm, the height of the captain, in feet above sea level, is given by the function

\[
h(t) = 15 \cos (kt) + c
\]

where \( k \) and \( c \) are nonzero constants.

a. [3 points] Find a formula for \( v(t) \), the vertical velocity of the captain, in feet per second, as a function of \( t \). The constants \( k \) and \( c \) may appear in your answer.

Solution: The velocity is the derivative of the height function, so we compute

\[
v(t) = h'(t) = -15k \sin (kt) .
\]

Notice that the Chain Rule gives us a factor of \( k \) out front, and since \( c \) is an additive constant, it disappears when we take the derivative.

Notice also that \( v(t) = \frac{dh}{dt} \) does indeed have units of feet per second, as required.

Answer: \( v(t) = -15k \sin (kt) \)

b. [2 points] Find a formula for \( v'(t) \). The constants \( k \) and \( c \) may appear in your answer.

Answer: \( v'(t) = -15k^2 \cos (kt) \)

c. [3 points] What is the maximum vertical acceleration experienced by the captain? The constants \( k \) and \( c \) may appear in your answer. You do not need to justify your answer or show work. Remember to include units.

Solution: The acceleration is just the derivative of the velocity function, which was just computed in the previous part.

Since \( v'(t) = -15k^2 \cos (kt) \) is sinusoidal with midline 0 and amplitude \( 15k^2 \), the maximum value it achieves is \( 15k^2 \).

Since \( v'(t) = \frac{dv}{dt} \), the units on the acceleration are feet per second per second, or feet per second squared.

Answer: Max vertical acceleration: \( 15k^2 \text{ ft/s}^2 \)