4. [8 points] A ship's captain is standing on the deck while sailing through stormy seas. The rough waters toss the ship about, causing it to rise and fall in a sinusoidal pattern. Suppose that t seconds into the storm, the height of the captain, in feet above sea level, is given by the function

$$h(t) = 15\cos(kt) + c$$

where k and c are nonzero constants.

a. [3 points] Find a formula for $v(t)$, the vertical velocity of the captain, in feet per second, as a function of t. The constants k and c may appear in your answer.

Solution: The velocity is the derivative of the height function, so we compute

$$v(t) = h'(t) = -15k\sin(kt).$$

Notice that the Chain Rule gives us a factor of k out front, and since c is an additive constant, it disappears when we take the derivative.
Notice also that $v(t) = \frac{dh}{dt}$ does indeed have units of feet per second, as required.

Answer: $v(t) =$ $-15k\sin(kt)$

b. [2 points] Find a formula for $v'(t)$. The constants k and c may appear in your answer.

Answer: $v'(t) =$ $-15k^2\cos(kt)$

c. [3 points] What is the maximum vertical acceleration experienced by the captain? The constants k and c may appear in your answer. You do not need to justify your answer or show work. *Remember to include units.*

Solution: The acceleration is just the derivative of the velocity function, which was just computed in the previous part.
Since $v'(t) = -15k^2\cos(kt)$ is sinusoidal with midline 0 and amplitude $15k^2$, the maximum value it achieves is $15k^2$.
Since $v'(t) = \frac{dv}{dt}$, the units on the acceleration are feet per second per second, or feet per second squared.

Answer: Max vertical acceleration: $15k^2$ **ft/s^2**