7. [5 points] Let

$$s(t) = \begin{cases} 5t^2 & \text{if } t \le 3 \\ p + c(t-3) & \text{if } t > 3 \end{cases}$$

be a differentiable function, where p and c are constants.

a. [3 points] Find the values of p and c.

Solution: Since $s(t)$ is differentiable, it is also continuous. By continuity, the two parts must agree at $t = 3$, so we have

$$5 \cdot 3^2 = p + c(3-3) = p,$$

or $p = 45$.

By differentiability, $s'(t)$ must exist at $t = 3$. For $t < 3$, we have $s'(t) = 10t$, and for $t > 3$, we have $s'(t) = c$. To be differentiable at $t = 3$, these two must agree at $t = 3$, so we have

$$10 \cdot 3 = c,$$

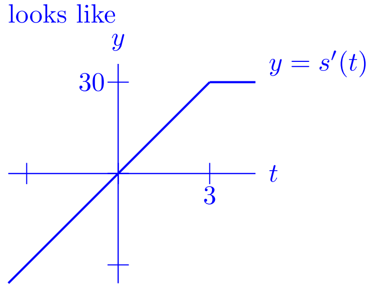
or $c = 30$.

Answer: $p =$ 45 and $c =$ 30

b. [2 points] Is $s'(t)$ differentiable at $t = 3$?
To receive any credit on this question, you must justify your answer.

Solution: No. We saw above that $s'(t) = \begin{cases} 10t & \text{if } t \le 3 \\ 30 & \text{if } t > 3 \end{cases}$.

The graph of $y = s'(t)$ therefore looks like



which has a sharp corner at $t = 3$.

8. [6 points] Find a formula for $\frac{dy}{dx}$ for the implicit function $ax^2 + xy^2 + b\ln y = c$.
The constants a, b, and c may appear in your answer.

Solution: Applying $\frac{d}{dx}$ to both sides of the given equation, we have

$$2ax + y^2 + 2xy\frac{dy}{dx} + \frac{b}{y}\frac{dy}{dx} = 0.$$

Collecting all the terms involving $\frac{dy}{dx}$ on one side and then factoring it out, we find

$$\frac{dy}{dx}\left(2xy + \frac{b}{y}\right) = -2ax - y^2$$

and hence

$$\frac{dy}{dx} = \frac{-2ax - y^2}{2xy + \frac{b}{y}}.$$

Answer: $\frac{dy}{dx} = \frac{-2ax - y^2}{2xy + \frac{b}{y}}$