7. [5 points] Let 
\[ s(t) = \begin{cases} 
5t^2 & \text{if } t \leq 3 \\
p + c(t - 3) & \text{if } t > 3 
\end{cases} \]
be a differentiable function, where \( p \) and \( c \) are constants.

a. [3 points] Find the values of \( p \) and \( c \).

\[ \text{Solution: Since } s(t) \text{ is differentiable, it is also continuous. By continuity, the two parts must agree at } t = 3, \text{ so we have} \]
\[ 5 \cdot 3^2 = p + c(3 - 3) = p, \]
or \( p = 45 \).

By differentiability, \( s'(t) \) must exist at \( t = 3 \). For \( t < 3 \), we have \( s'(t) = 10t \), and for \( t > 3 \), we have \( s'(t) = c \). To be differentiable at \( t = 3 \), these two must agree at \( t = 3 \), so we have
\[ 10 \cdot 3 = c, \]
or \( c = 30 \).

\[ \text{Answer: } p = \boxed{45} \text{ and } c = \boxed{30} \]

b. [2 points] Is \( s'(t) \) differentiable at \( t = 3 \)?

To receive any credit on this question, you must justify your answer.

\[ \text{Solution: No. We saw above that } s'(t) = \begin{cases} 
10t & \text{if } t \leq 3 \\
30 & \text{if } t > 3 
\end{cases} \]

The graph of \( y = s'(t) \) therefore looks like

\[ \text{which has a sharp corner at } t = 3. \]

8. [6 points] Find a formula for \( \frac{dy}{dx} \) for the implicit function \( ax^2 + xy^2 + b \ln y = c \).

The constants \( a, b, \) and \( c \) may appear in your answer.

\[ \text{Solution: Applying } \frac{d}{dx} \text{ to both sides of the given equation, we have} \]
\[ 2ax + y^2 + 2xy \frac{dy}{dx} + \frac{b}{y} \frac{dy}{dx} = 0. \]

Collecting all the terms involving \( \frac{dy}{dx} \) on one side and then factoring it out, we find
\[ \frac{dy}{dx} \left( 2xy + \frac{b}{y} \right) = -2ax - y^2 \]
and hence
\[ \frac{dy}{dx} = \frac{-2ax - y^2}{2xy + \frac{b}{y}}. \]

\[ \text{Answer: } \frac{dy}{dx} = \frac{-2ax - y^2}{2xy + \frac{b}{y}}. \]