9. [10 points] After a long, cold winter, a ship's captain sails across Lake Michigan to Chicago. Upon arrival, the captain hosts a party on board to celebrate the arrival of spring. The party begins at exactly 6 pm and ends at exactly midnight. Let $N(t)$ be the noise level, in decibels, of the ship captain's party t hours after it begins. During the party, a formula for $N(t)$ is given by

$$N(t) = 0.5t^4 - 4t^3 + 7t^2 + 60.$$

a. [8 points] Find the exact values of t that minimize and maximize $N(t)$ on the interval $[0, 6]$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema.

Solution: By the Extreme Value Theorem, there will be both a global minimum and a global maximum, and they will occur at either the end points or the critical points. So we begin by finding the critical points.
We have $N'(t) = 2t^3 - 12t^2 + 14t = 2t(t^2 - 6t + 7)$. There are no points where this is undefined, so our only critical points are at the zeros. We immediately see that $t = 0$ is a zero; for the others, we use the Quadratic Formula to find two more zeros at

$$t = \frac{6 \pm \sqrt{6^2 - 28}}{2} = 3 \pm \sqrt{2}.$$

Our critical points are therefore $t = 0$, $3 - \sqrt{2}$, and $3 + \sqrt{2}$.
To determine the global extrema, then, we compare the values of $N(t)$ at all critical points and end points of our interval:

t	0	$3-\sqrt{2}$	$3+\sqrt{2}$	6
$N(t)$	60	64.8	42.2	96

Since the smallest value of $N(t)$ occurs at $t = 3 + \sqrt{2}$, this is our global minimum, and since the largest value of $N(t)$ occurs at $t = 6$, this is our global maximum.

(For each answer blank below, write NONE in the answer blank if appropriate.)

Answer: Global min(s) at exactly $t =$ $3 + \sqrt{2}$

Answer: Global max(es) at exactly $t =$ 6

b. [2 points] How loud does the captain's party get? *Remember to include units.*

Solution: We just saw that the maximum value of this function occurs at $t = 6$, when $N(6) = 96$. Therefore, the loudest the captain's party gets is 96 decibels.

Answer: 96 **decibels**