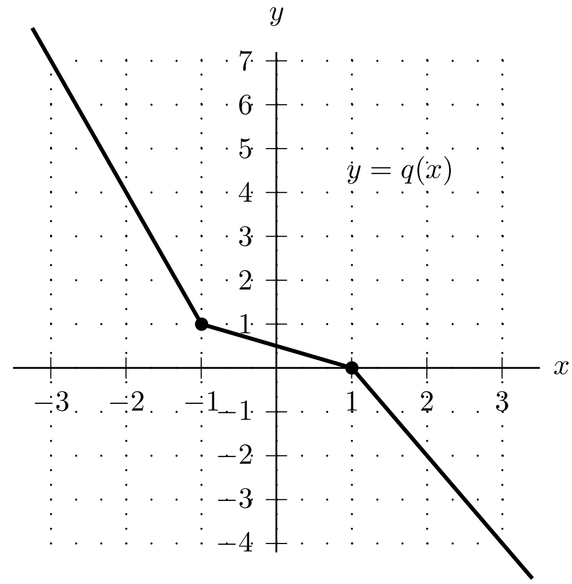
1. [11 points]

Shown to the right is the graph of an invertible piecewise linear function $q(x)$. Note that the graph passes through the points $(-3,7)$, $(-1,1)$, $(1,0)$, and $(3,-4)$.



You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

Find the exact value of each of the quantities below. If there is not enough information provided to find the value, write "NOT ENOUGH INFO". If the value does not exist, write "DOES NOT EXIST".

a. [2 points] Let $r(x) = q^{-1}(x)$. Find $r'(2)$.

Solution: $r'(x) = \dfrac{1}{q'(q^{-1}(x))}$ so $r'(2) = \dfrac{1}{q'(q^{-1}(2))} = \dfrac{1}{-3} = -\dfrac{1}{3}$.

Answer: $r'(2) = -\dfrac{1}{3}$

b. [3 points] Let $w(x) = \dfrac{x}{q(x+1)}$. Find $w'(-2)$.

Solution: By the quotient and chain rules, $w'(x) = \frac{q(x+1)-xq'(x+1)}{(q(x+1))^2}$ (where these quantities are defined). q' is not differentiable at $x = -1$, so $q'(x+1)$ is not defined at $x = -2$. (If $w'(-2)$ were to exist, then since $q(x+1) = \frac{w(x)}{x}$, we would have $q'(-1) = q'(-2+1) = \frac{(-2)w'(-2)-w(-2)}{(-2)^2}$.)

Answer: $w'(-2) =$ DOES NOT EXIST

c. [3 points] Let $v(x) = xq(\sin x)$. Find $v'(\pi)$.

Solution: By the product and chain rules we have $v'(x) = xq'(\sin x)\cos x + q(\sin x)$. So $v'(\pi) = \pi q'(\sin\pi)\cos\pi + q(\sin\pi) = \pi q'(0)(-1) + q(0) = \pi(-1/2)(-1) + (1/2) = \frac{\pi+1}{2}$.

Answer: $v'(\pi) = \dfrac{\pi+1}{2}$

d. [3 points] Let $j(x) = \ln(q(2x))$. Find $j'(-1)$.

Solution: By the chain rule, we have

$$j'(x) = \frac{1}{q(2x)} \cdot q'(2x) \cdot 2 = \frac{2q'(2x)}{q(2x)} \quad \text{so} \quad j'(-1) = \frac{2q'(-2)}{q(-2)} = \frac{2(-3)}{4} = -\frac{3}{2}.$$

Answer: $j'(-1) = -\dfrac{3}{2}$