2. [13 points] A function $g(x)$ and its derivative are given by

$$g(x) = \frac{x^3 + 34x^2 + 732x + 5400}{(x+30)^4} \quad \text{and} \quad g'(x) = \frac{-(x-6)^2(x-10)}{(x+30)^5}.$$

a. [8 points] Find all critical points of $g(x)$ and all values of x at which $g(x)$ has a local extremum. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

Solution: To find critical points we (i) solve for x in the equation $g'(x) = 0$ and (ii) look for points in the domain of the function where the derivative is undefined. The critical points occur at $x = 6$ and $x = 10$. Note that $x = -30$ is not a critical point since it is not in the domain of the function.

To test whether each of these critical points is a local extremum, we consider the sign of $g'(x)$ on the intervals on each side of the critical points and then apply the First Derivative Test. Note that $(x-6)^2$ is positive for all $x \neq 6$, and $(x+30)^5$ is positive for all $x > -30$. Finally, note that $x - 10$ is positive for $x > 10$ and negative for $x < 10$. We summarize the resulting signs of the derivative in the table below.

Interval	$-30 < x < 6$	$6 < x < 10$	$x > 10$
Sign of $g'(x)$	$\frac{-\cdot+\cdot-}{+} = +$	$\frac{-\cdot+\cdot-}{+} = +$	$\frac{-\cdot+\cdot+}{+} = -$

We see that $g(x)$ does not have a local extremum at $x = 6$ and that, by the First Derivative Test, $g(x)$ has a local maximum at $x = 10$.

(For each answer blank below, write NONE in the answer blank if appropriate.)

Answer: critical point(s) at $x =$ 6, 10

Answer: local min(s) at $x =$ **NONE**

Answer: local max(es) at $x =$ 10

b. [5 points] Find the values of x that minimize and maximize $g(x)$ on the interval $[0, \infty)$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema.

Solution: The only local extremum is the local maximum at $x = 10$ so since g is continuous on the interval $[0, \infty)$,this must also be the global max. (Alternatively, note that from part (a), we know that $g(x)$ is increasing for $0 \leq x \leq 10$ and decreasing for $x \geq 10$, so $g(10)$ must be the global maximum value of $g(x)$ on the interval $[0, \infty)$.) To check for a global min we need to consider the ends of the interval. Now, $g(0) = \frac{1}{150}$ and $\lim_{x\to\infty} g(x) = 0$, which is less than $\frac{1}{150}$. This implies that there is no global minimum of $g(x)$ on the interval $[0, \infty)$.

(For each answer blank below, write NONE in the answer blank if appropriate.)

Answer: global min(s) at $x =$ **NONE**

Answer: global max(es) at $x =$ 10