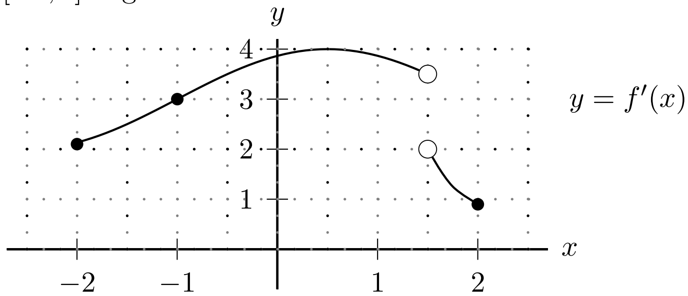
3. [8 points] Suppose $f(x)$ is a function that is continuous on the interval $[-2, 2]$. The graph of $f'(x)$ on the interval $[-2, 2]$ is given below.



a. [3 points] Let $L(x)$ be the local linearization of $f(x)$ at $x = -1$. Using the fact that $f(-1) = 4$, write a formula for $L(x)$.

Solution: $f(-1) = 4$ and $f'(-1) = 3$, so $L(x) = 4 + 3(x - (-1)) = 4 + 3(x + 1)$.

Answer: $L(x) =$ $4 + 3(x + 1)$ **or** $3x + 7$

b. [2 points] Use your formula for $L(x)$ to approximate $f(-0.5)$.

Solution: Since -0.5 is close to -1 we have

$$f(-0.5) \approx L(-0.5) = 4 + 3(-0.5 + 1) = 43(0.5) = 5.5.$$

Answer: $f(-0.5) \approx$ 5.5

c. [3 points] Is your answer from part (b) an overestimate or an underestimate of the actual value of $f(-0.5)$? Justify your answer.

Circle one: overestimate **[underestimate]** CANNOT BE DETERMINED

Justification:

Solution: The function $f'(x)$ is increasing between -2 and 0 so $f(x)$ is concave up over this interval. Therefore the tangent line to the graph of $f(x)$ at $x = -1$ lies below the graph of $f(x)$ between $x = -2$ and $x = 0$. In particular, the local linearization $L(x)$ of $f(x)$ at $x = -1$ gives an underestimate of f on that interval.