

4. [11 points] Elphaba has found a corrupt prison guard, Mert, to sell her metal piping to use to dig a tunnel out of the prison. Mert can sell Elphaba steel piping and copper piping, and he provides the following information.
- The number of kilograms (kg) of soil that Elphaba can dig with steel piping is proportional to the number of centimeters (cm) of steel piping that she buys. She can dig 50 kg of soil per cm of steel piping, and her cost (in dollars) of buying x cm of steel piping is given by $A(x) = x^2 + x$.
 - The number of kilograms (kg) of soil that Elphaba can dig with copper piping is proportional to the number of centimeters (cm) of copper piping that she buys. She can dig 30 kg of soil per cm of copper piping, and her cost (in dollars) of buying y cm of copper piping is given by $B(y) = 2y$.
- a. [1 point] How many kilograms of soil can Elphaba dig with x cm of steel piping?

Answer: _____ $50x$

For parts (b)-(d) below, suppose Elphaba buys w cm of steel piping and k cm of copper piping and that this is exactly the right amount of piping so that she can dig through 2700 kg of soil to dig her escape tunnel.

- b. [3 points] Write a formula for k in terms of w .

Solution: With w cm of steel piping, she can dig through $50w$ kg of soil and with k cm of copper piping, she can dig through $30k$ kg of soil. So $50w + 30k = 2700$, and solving for k , we find $k = \frac{2700-50w}{30}$.

Answer: $k = \frac{2700 - 50w}{30} = 90 - \frac{5}{3}w$

- c. [4 points] Let $T(w)$ be the total cost (in dollars) of all the piping Elphaba buys to dig her escape tunnel. Find a formula for the function $T(w)$. The variable k and the function names A and B should not appear in your answer.
(Note that $T(w)$ is the function one would use to minimize Elphaba's costs. You should not do the optimization in this case.)

Solution: $T(w) = A(w) + B\left(\frac{2700 - 50w}{30}\right) = w^2 + w + 2\left(\frac{2700 - 50w}{30}\right)$.

Answer: $T(w) = w^2 + w + 2\left(\frac{2700 - 50w}{30}\right)$ or $w^2 - \frac{7}{3}w + 18$

- d. [3 points] What is the domain of $T(w)$ in the context of this problem?

Solution: In the context of this problem, the smallest possible value of w is 0, which would occur if Elphaba were to buy only copper piping. The largest possible value of w would occur if Elphaba were to buy only steel piping. In that case, $50w = 2700$ so $w = \frac{2700}{50} = 54$. In the context of this problem, the domain of $T(w)$ consists of all values of w between 0 and 54, i.e. the interval $[0, 54]$.

Answer: _____ $[0, 54]$