6. [14 points] Let p be a function such that $p''(x)$ is defined for all real numbers x. A table of some values of $p'(x)$ is given below.

x	-9	-5	-1	3	7	11
$p'(x)$	-3	0	-4	0	2	1

Assume that p' is either always strictly decreasing or always strictly increasing between consecutive values of x shown in the table.

For each of the questions below, circle ALL of the appropriate choices. If none of the choices are correct, circle NONE OF THESE.

a. [2 points] At which, if any, of the following values of x does $p(x)$ definitely have a local maximum in the interval $-9 < x < 11$?

-5 -1 3 7 NONE OF THESE

Solution: The only critical points of $p(x)$ in this interval are at $x = -5$ and $x = 3$. We now classify these critical points. $p(x)$ does not have a local extremum at $x = -5$ since $p'(x)$ is negative both immediately before and after $x = -5$. $p(x)$ has a local minimum at $x = 3$ since the sign of $p'(x)$ changes from negative to positive there.

b. [2 points] At which, if any, of the following values of x does $p(x)$ definitely attain its global minimum on the interval $-9 \leq x \leq 11$?

-9 -5 -1 3 7 11 NONE OF THESE

Solution: $p(x)$ is decreasing for $-9 \leq x \leq 3$ and increasing for $3 \leq x \leq 11$.

c. [2 points] At which, if any, of the following values of x does $p'(x)$ (the derivative of $p(x)$) definitely attain its global maximum on the interval $-9 \leq x \leq 11$?

-9 -5 -1 3 7 11 NONE OF THESE

Solution: Since $p'(x)$ is always increasing of decreasing between points in the table, the global max must occur at one of the values of x in the table. Realizing this, we need to choose the value of x in the table that gives the largest value of $p'(x)$, which is $x = 7$.

d. [3 points] On which of the following intervals is $p(x)$ definitely always concave up?

$-9 < x < -5$ $-5 < x < -1$ $-1 < x < 3$ $3 < x < 7$ $7 < x < 11$ NONE OF THESE

Solution: The function $p(x)$ is concave up whenever the derivative $p'(x)$ is increasing.

e. [3 points] At which, if any, of the following values of x does $p(x)$ definitely have an inflection point in the interval$-9 < x < 11$?

-5 -1 3 7 NONE OF THESE

Solution: We are looking for places where $p'(x)$ changes from increasing to decreasing or vice versa.

f. [2 points] Which, if any, of the following must be true?

$p''(7) \geq p''(-3)$ $p''(7) = p''(-3)$ $p''(7) \leq p''(-3)$ NONE OF THESE

Solution: The function $p'(x)$ is increasing between 3 and 7 and decreasing between 7 and 11 so $p''(7) = 0$. The function $p'(x)$ is decreasing between -5 and -1, so $p''(-3) \leq 0$.