

9. [9 points] Elphaba and Walt are planning to break out of prison. They would like to escape no later than 20 hours after devising their plan, and they would like to attempt their escape during the noisiest part of the day. Let  $N(t)$  be the noise level (in decibels) in the prison  $t$  hours after Elphaba and Walt have devised their escape plan. On the interval  $[0, 20]$ , a formula for  $N(t)$  is given by

$$N(t) = 60 + 1.01^{p(t)} \quad \text{where} \quad p(t) = \frac{1}{3}t^3 - 9t^2 + 56t + 200.$$

- a. [8 points] Find the values of  $t$  that minimize and maximize  $N(t)$  on the interval  $[0, 20]$ . Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema.

*Solution:* Since  $N(t)$  is continuous on the interval  $[0, 20]$ , we can apply the Extreme Value Theorem and compare the values of  $N(t)$  at the critical points and endpoints of the interval.

We first need to find the critical points in this interval. Taking the derivative of  $N(t)$  and setting it equal to zero we have  $N'(t) = \ln(1.01)p'(t)(1.01)^{p(t)} = 0$  so critical points occur when  $0 = p'(t) = t^2 - 18t + 56$ . Solving we determine that the only critical points of  $N(t)$  occur at  $t = 4$  and  $t = 14$ , which are both in the interval  $[0, 20]$ .

To find the global extrema we need to evaluate the function at  $t = 0, 4, 14, 20$ . We find  $N(0) \approx 67.316$ ,  $N(4) \approx 80.0528$ ,  $N(14) \approx 63.819$  and  $N(20) \approx 106.874$ . Choosing the largest and smallest values, by the Extreme Value Theorem, we see that the global minimum occurs at  $t = 14$  and the global maximum occurs at  $t = 20$ .

(For each answer blank below, write NONE in the answer blank if appropriate.)

**Answer:** global min(s) at  $t =$  \_\_\_\_\_ 14

**Answer:** global max(es) at  $t =$  \_\_\_\_\_ 20

- b. [1 point] As mentioned above, Elphaba and Walt would like to escape no later than 20 hours after devising their plan, and they would like to escape during the noisiest part of the day. When should Elphaba and Walt attempt their escape?

**Answer:** They should try to escape \_\_\_\_\_ 20 hours after devising their plan.