Suppose \( H \) is a differentiable function such that \( H'(w) \) is also differentiable for \( 0 < w < 10 \). Several values of \( H(w) \) and of its first and second derivatives are given in the table on the right.

<table>
<thead>
<tr>
<th>( w )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H(w) )</td>
<td>6.3</td>
<td>5.4</td>
<td>5.2</td>
<td>4.8</td>
<td>0.7</td>
</tr>
<tr>
<td>( H'(w) )</td>
<td>−1.5</td>
<td>−0.4</td>
<td>−0.1</td>
<td>−0.6</td>
<td>−2.1</td>
</tr>
<tr>
<td>( H''(w) )</td>
<td>1.6</td>
<td>0.9</td>
<td>0</td>
<td>−0.8</td>
<td>−0.4</td>
</tr>
</tbody>
</table>

Assume that between each pair of consecutive values of \( w \) shown in the table, each of \( H'(w) \) and \( H''(w) \) is either always strictly decreasing or always strictly increasing.

a. [3 points] Use an appropriate linear approximation to estimate \( H(5.2) \).

\[
\text{Answer: } H(5.2) \approx \quad \text{[Value]} \]

b. [5 points] Let \( J(w) \) be the local linearization of \( H \) near \( w = 2 \), and let \( K(w) \) be the local linearization of \( H \) near \( w = 3 \). Which of the following statements must be true? Circle all of the statements that must be true, or circle "NONE OF THESE".

- \( J(2) > H(2) \)
- \( J(2) = H(2) \)
- \( J(2) < H(2) \)
- \( J'(2) > H'(2) \)
- \( J'(2) = H'(2) \)
- \( J'(2) < H'(2) \)
- \( K(3.5) > H(3.5) \)
- \( K(3.5) = H(3.5) \)
- \( K(3.5) < H(3.5) \)
- \( K'(3.5) > H'(3.5) \)
- \( K'(3.5) = H'(3.5) \)
- \( K'(3.5) < H'(3.5) \)
- NONE OF THESE

c. [3 points] Use the quadratic approximation of \( H(w) \) at \( w = 1 \) to estimate \( H(0.9) \).

(Recall that a formula for the quadratic approximation \( Q(x) \) of a function \( f(x) \) at \( x = a \) is \( Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2 \).

\[
\text{Answer: } H(0.9) \approx \quad \text{[Value]} \]

d. [3 points] Consider the function \( N \) defined by \( N(w) = H(2w^2 - 10) \), and let \( L(w) \) be the local linearization of \( N(w) \) at \( w = 3 \). Find a formula for \( L(w) \). Your answer should not include the function names \( N \) or \( H \).

\[
\text{Answer: } L(w) = \quad \text{[Value]} \]