8. [14 points]
Suppose H is a differentiable function such that $H'(w)$ is also differentiable for $0 < w < 10$. Several values of $H(w)$ and of its first and second derivatives are given in the table on the right.

w	1	2	3	5	8
$H(w)$	6.3	5.4	5.2	4.8	0.7
$H'(w)$	-1.5	-0.4	-0.1	-0.6	-2.1
$H''(w)$	1.6	0.9	0	-0.8	-0.4

Assume that between each pair of consecutive values of w shown in the table, each of $H'(w)$ and $H''(w)$ is either always strictly decreasing or always strictly increasing. *Remember to show your work carefully.*

a. [3 points] Use an appropriate linear approximation to estimate $H(5.2)$.

Answer: $H(5.2) \approx$ ____________________

b. [5 points] Let $J(w)$ be the local linearization of H near $w = 2$, and let $K(w)$ be the local linearization of H near $w = 3$. Which of the following statements must be true?
Circle all of the statements that must be true, or circle "NONE OF THESE".

$J(2) > H(2)$	$J(2.5) > H(2.5)$	$K(3.5) > H(3.5)$
$J(2) = H(2)$	$J(2.5) = H(2.5)$	$K(3.5) = H(3.5)$
$J(2) < H(2)$	$J(2.5) < H(2.5)$	$K(3.5) < H(3.5)$
$J'(2) > H'(2)$	$K(2.5) > H(2.5)$	$K'(3.5) > H'(3.5)$
$J'(2) = H'(2)$	$K(2.5) = H(2.5)$	$K'(3.5) = H'(3.5)$
$J'(2) < H'(2)$	$K(2.5) < H(2.5)$	$K'(3.5) < H'(3.5)$

NONE OF THESE

c. [3 points] Use the quadratic approximation of $H(w)$ at $w = 1$ to estimate $H(0.9)$.
(Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$.)

Answer: $H(0.9) \approx$ ____________________

d. [3 points] Consider the function N defined by $N(w) = H(2w^2 - 10)$, and let $L(w)$ be the local linearization of $N(w)$ at $w = 3$. Find a formula for $L(w)$. Your answer should not include the function names N or H.

Answer: $L(w) =$ ____________________