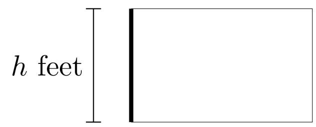
1. [7 points] Gertrude wants to enclose a rectangular region in her backyard. She wants to use high fencing (thick line), which costs \$200 per foot, for one side of the rectangle. For the remaining three sides, she wants to use normal fencing (thin line), which costs \$75 per foot. Let $A(h)$ be the area (in square feet) of the region enclosed by the fence if h is the length (in feet) of the side with high fencing and Gertrude spends \$3000 on fencing for the project.



a. [4 points] Find a formula for $A(h)$.

Solution: Let ℓ be the other sidelength of the rectangle. Then, the total cost of the fencing is

$$200h + 75(2\ell + h) = 275h + 150\ell.$$

If the total cost of fencing is \$3000, then

$$\begin{aligned} 275h + 150\ell &= 3000 \\ 150\ell &= 3000 - 275h \\ \ell &= 20 - \frac{11}{6}h. \end{aligned}$$

Hence,

$$A(h) = h\ell = 20h - \frac{11}{6}h^2.$$

Answer: $A(h) =$ $20h - \frac{11}{6}h^2$

b. [3 points] In the context of this problem, what is the domain of $A(h)$?

Solution: Note that $h > 0$, or else we would not have a rectangle. Note also that $\ell > 0$ (where ℓ is the other sidelength).
So since $275h + 150\ell = 3000$, we have $275h = 3000 - 150\ell < 3000$, so $h < \frac{3000}{275} = \frac{120}{11} \approx 10.91$. Hence, the domain of $A(h)$ is $0 < h < \frac{120}{11}$.

(Note that in this situation, it would also be okay to include the endpoints 0 and 3000/275, which correspond to the degenerate cases of a rectangle of length or width 0.)

Answer: Domain: **The interval** $\left(0, \frac{120}{11}\right)$ **(or** $\left[0, \frac{120}{11}\right]$**)**