3. [9 points] Consider the curve $\mathcal{C}$ defined by

$$\cos(ax - y) + x^2 + y^2 = b$$

where a and b are positive constants.

a. [5 points] For the curve $\mathcal{C}$, find a formula for $\frac{dy}{dx}$ in terms of x and y. The constants a and b may appear in your answer. To earn credit for this problem, you must compute this by hand and show every step of your work clearly.

Solution:
Implicit differentiation:

$$\frac{d}{dx}\left(\cos(ax-y) + x^2 + y^2\right) = \frac{d}{dx}(b)$$
$$\left(a - \frac{dy}{dx}\right)(-\sin(ax-y)) + 2x + 2y\frac{dy}{dx} = 0$$

Solving for $\frac{dy}{dx}$:

$$-a\sin(ax-y) + \frac{dy}{dx}(\sin(ax-y)) + 2x + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(\sin(ax-y) + 2y) = a\sin(ax-y) - 2x$$
$$\frac{dy}{dx} = \frac{a\sin(ax-y) - 2x}{\sin(ax-y) + 2y}$$

Answer: $\frac{dy}{dx} = \frac{a\sin(ax-y) - 2x}{\sin(ax-y) + 2y}$

b. [1 point] Let $a = 1$ and $b = 9$. Exactly one of the following points (x, y) lies on the curve $\mathcal{C}$. Circle that one point.

$(3, 0)$ $\boxed{(2, 2)}$ $(1, -1)$ (π, π) $(0, -9)$

c. [3 points] With $a = 1$ and $b = 9$ as above, find an equation for the tangent line to the curve $\mathcal{C}$ at the point you chose in part **b.**.

Solution: To find the slope of the tangent line, we evaluate $\frac{dy}{dx}$ at the point $(2, 2)$ to find

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,2)} = \frac{\sin(2-2) - 2(2)}{\sin(2-2) + 2(2)} = -1.$$

Hence, the equation for the tangent line is $y = 2 - 1(x - 2) = -x + 4$.

Answer: $y =$ $2 - (x - 2)$ **(or** $-x + 4$**)**