7. [7 points] Alicia decides to go for a run before completing her math homework. Let $g(m)$ be the time (in hours) that Alicia spends completing her math assignment if she runs m miles. Suppose that for $1.2 \leq m \leq 8$,

$$g(m) = 2m - 12.2\ln(m) + 15 - \frac{14.4}{m}.$$

Note that on this interval, the derivative of g is given by the formula

$$g'(m) = \frac{2(m-4.5)(m-1.6)}{m^2}.$$

a. [5 points] Find all values of m that maximize and minimize the function $g(m)$ on the interval $1.2 \leq m \leq 8$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema.

Solution: Since g is continuous on the closed interval $[1.2, 8]$, by the Extreme Value Theorem g definitely attains a global maximum and global minimum on the interval, and it suffices to compare the values of $g(m)$ at the critical points and endpoints of the interval.
Notice that the critical points of $g(m)$ in the interval $1.2 \leq m \leq 8$ are at $m = 1.6$ and $m = 4.5$. Hence, we need to check the value of $g(m)$ at $m = 1.2, 1.6, 4.5, 8$:

$$g(1.2) = 2(1.2) - 12.2\ln(1.2) + 15 - \frac{14.4}{1.2} \approx 3.176$$
$$g(1.6) = 2(1.6) - 12.2\ln(1.6) + 15 - \frac{14.4}{1.6} \approx 3.466$$
$$g(4.5) = 2(4.5) - 12.2\ln(4.5) + 15 - \frac{14.4}{4.5} \approx 2.450$$
$$g(8) = 2(8) - 12.2\ln(8) + 15 - \frac{14.4}{8} \approx 3.831$$

Thus, we can see that $g(m)$ achieves its maximum on the interval at $m = 8$, and $g(m)$ achieves its minimum on the interval at $m = 4.5$.

For each answer blank below, write "NONE" if appropriate.

Answer: Global max(es) at $m=$ 8

Answer: Global min(s) at $m=$ 4.5

b. [2 points] Assuming that Alicia runs at least 1.2 miles and at most 8 miles, what is the shortest amount of time Alicia could spend completing her homework?
Remember to include units.

Solution: As we saw in part **a.**, under those assumptions, the shortest amount of time Alicia could spend completing her homework is $g(4.5) \approx 2.450$ hours.

Answer: Shortest time: 2.450 **hours**