8. [14 points]
Suppose $H$ is a differentiable function such that $H'(w)$ is also differentiable for $0 < w < 10$. Several values of $H(w)$ and of its first and second derivatives are given in the table on the right.

Assume that between each pair of consecutive values of $w$ shown in the table, each of $H'(w)$ and $H''(w)$ is either always strictly decreasing or always strictly increasing. Remember to show your work carefully.

a. [3 points] Use an appropriate linear approximation to estimate $H(5.2)$.

**Solution:** For $w$ near 5, local linearization gives $H(w) \approx H(5) + H'(5)(w - 5)$, so

$$H(5.2) \approx H(5) + H'(5)(5.2 - 5) = 4.8 - 0.6(0.2) = 4.8 - 0.12 = 4.68.$$  

**Answer:** $H(5.2) \approx 4.68$

b. [5 points] Let $J(w)$ be the local linearization of $H$ near $w = 2$, and let $K(w)$ be the local linearization of $H$ near $w = 3$. Which of the following statements must be true? Circle all of the statements that must be true, or circle “NONE OF THESE”.

- $J(2) > H(2)$
- $J(2.5) > H(2.5)$
- $K(3.5) > H(3.5)$
- $J(2) = H(2)$
- $J(2.5) = H(2.5)$
- $K(3.5) = H(3.5)$
- $J(2) < H(2)$
- $J(2.5) < H(2.5)$
- $K(3.5) < H(3.5)$
- $J'(2) > H'(2)$
- $K(2.5) > H(2.5)$
- $K'(3.5) > H'(3.5)$
- $J'(2) = H'(2)$
- $K(2.5) = H(2.5)$
- $K'(3.5) = H'(3.5)$
- $J'(2) < H'(2)$
- $K(2.5) < H(2.5)$
- $K'(3.5) < H'(3.5)$
- NONE OF THESE

c. [3 points] Use the quadratic approximation of $H(w)$ at $w = 1$ to estimate $H(0.9)$. (Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$.)

**Solution:** Let $Q(w)$ be the quadratic approximation of $H(w)$ at $w = 1$.

Then $Q(w) = H(1) + H'(1)(w-1) + \frac{H''(1)}{2}(w-1)^2 = 6.3 - 1.5(w-1) + \frac{16}{2}(w-1)^2 = 6.3 + 15 + 0.008 = 6.458$.

**Answer:** $H(0.9) \approx 6.458$

d. [3 points] Consider the function $N$ defined by $N(w) = H(2w^2 - 10)$, and let $L(w)$ be the local linearization of $N(w)$ at $w = 3$. Find a formula for $L(w)$. Your answer should not include the function names $N$ or $H$.

**Solution:** We know that $L(w) = N(3) + N'(3)(w-3)$.  
Note that $N(3) = H(2(3^2) - 10) = H(8) = 0.7$,  
To find $N'(3)$, we apply the Chain Rule. In particular, $N'(w) = (4w) \cdot H'(2w^2 - 10)$, so $N'(3) = (4 \cdot 3) \cdot H'(2(3^2) - 10) = 12H'(8) = 12(-2.1) = -25.2$.  
Therefore, $L(w) = N(3) + N'(3)(w-3) = 0.7 - 25.2(w-3)$.

**Answer:** $L(w) = 0.7 - 25.2(w-3)$