8. [14 points]
Suppose H is a differentiable function such that $H'(w)$ is also differentiable for $0 < w < 10$. Several values of $H(w)$ and of its first and second derivatives are given in the table on the right.

w	1	2	3	5	8
$H(w)$	6.3	5.4	5.2	4.8	0.7
$H'(w)$	-1.5	-0.4	-0.1	-0.6	-2.1
$H''(w)$	1.6	0.9	0	-0.8	-0.4

Assume that between each pair of consecutive values of w shown in the table, each of $H'(w)$ and $H''(w)$ is either always strictly decreasing or always strictly increasing.
Remember to show your work carefully.

a. [3 points] Use an appropriate linear approximation to estimate $H(5.2)$.

Solution: For w near 5, local linearization gives $H(w) \approx H(5) + H'(5)(w-5)$, so

$$H(5.2) \approx H(5) + H'(5)(5.2-5) = 4.8 - 0.6(0.2) = 4.8 - 0.12 = 4.68.$$

Answer: $H(5.2) \approx$ 4.68

b. [5 points] Let $J(w)$ be the local linearization of H near $w = 2$, and let $K(w)$ be the local linearization of H near $w = 3$. Which of the following statements must be true? Circle all of the statements that must be true, or circle "NONE OF THESE".

$J(2) > H(2)$	$J(2.5) > H(2.5)$	**[$K(3.5) > H(3.5)$]**
[$J(2) = H(2)$]	$J(2.5) = H(2.5)$	$K(3.5) = H(3.5)$
$J(2) < H(2)$	**[$J(2.5) < H(2.5)$]**	$K(3.5) < H(3.5)$
$J'(2) > H'(2)$	$K(2.5) > H(2.5)$	**[$K'(3.5) > H'(3.5)$]**
[$J'(2) = H'(2)$]	$K(2.5) = H(2.5)$	$K'(3.5) = H'(3.5)$
$J'(2) < H'(2)$	**[$K(2.5) < H(2.5)$]**	$K'(3.5) < H'(3.5)$

NONE OF THESE

c. [3 points] Use the quadratic approximation of $H(w)$ at $w = 1$ to estimate $H(0.9)$. (Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$.)

Solution: Let $Q(w)$ be the quadratic approximation of $H(w)$ at $w = 1$.
Then $Q(w) = H(1) + H'(1)(w-1) + \frac{H''(1)}{2}(w-1)^2 = 6.3 - 1.5(w-1) + \frac{1.6}{2}(w-1)^2$.
So, $H(0.9) \approx Q(0.9) = 6.3 - 1.5(0.9-1) + \frac{1.6}{2}(0.9-1)^2 = 6.3 + 0.15 + 0.008 = 6.458$.

Answer: $H(0.9) \approx$ 6.458

d. [3 points] Consider the function N defined by $N(w) = H(2w^2 - 10)$, and let $L(w)$ be the local linearization of $N(w)$ at $w = 3$. Find a formula for $L(w)$. Your answer should not include the function names N or H.

Solution: We know that $L(w) = N(3) + N'(3)(w-3)$.
Note that $N(3) = H(2(3^2) - 10) = H(8) = 0.7$.
To find $N'(3)$, we apply the Chain Rule. In particular, $N'(w) = (4w) \cdot H'(2w^2 - 10)$, so $N'(3) = (4 \cdot 3) \cdot H'(2(3^2) - 10) = 12H'(8) = 12(-2.1) = -25.2$.
Therefore, $L(w) = N(3) + N'(3)(w-3) = 0.7 - 25.2(w-3)$.

Answer: $L(w) =$ $0.7 - 25.2(w-3)$