2. [10 points] Let $R(x)$ be a polynomial whose first and second derivatives are given below.

$$R'(x) = (x-1)^7(x+2)^4 \qquad \text{and} \qquad R''(x) = (11x+10)(x-1)^6(x+2)^3$$

a. [6 points] Find the x-coordinates of the inflection points of $R(x)$. Use calculus to find and justify your answers, and show enough evidence to demonstrate that you have found them all. Write NONE if the function $R(x)$ has no points of inflection.

Solution: Potential inflection points: Since $R''(x)$ is defined for all x, we want all the solutions to $R''(x) = 0$, that is $x = -\frac{10}{11}$, $x = 1$ and $x = -2$.
Looking at the signs of $R''(x)$ around these points:

$$R''(-3) = (-)(+)(-) = +$$
$$R''(-1) = (-)(+)(+) = -$$
$$R''(0) = (+)(+)(+) = +$$
$$R''(2) = (+)(+)(+) = +$$

OR you can compute the values of R"(x) around these points

$$R''(-3) = 94208$$
$$R''(-1) = -64$$
$$R''(0) = 80$$
$$R''(2) = 2048$$

Since the sign of $R''(x)$ only changes at $x = -2$ and $x = -\frac{10}{11}$ then the inflection points of $R(x)$ are at $x = -2, -\frac{10}{11}$

b. [4 points] Find the quadratic approximation $G(x)$ of $R(x)$ at the point $(-1, 5)$ on the graph of $R(x)$. Show all your work.

Solution: $G(x) = R(-1) + R'(-1)(x+1) + \dfrac{R''(-1)}{2}(x+1)^2$ where

$R(-1) = 5$ since $(-1, 5)$ is a point on the graph of $R(x)$.

$$R'(-1) = (-1-1)^7(-1+2)^4 = -128$$
$$R''(-1) = (11(-1)+10)(-1-1)^6(-1+2)^3 = -64$$

Hence $G(x) = 5 - 128(x+1) - 32(x+1)^2$