2. [10 points] Let $R(x)$ be a polynomial whose first and second derivatives are given below.

$$R'(x) = (x - 1)^7(x + 2)^4 \quad \text{and} \quad R''(x) = (11x + 10)(x - 1)^6(x + 2)^3$$

a. [6 points] Find the $x$-coordinates of the inflection points of $R(x)$. Use calculus to find and justify your answers, and show enough evidence to demonstrate that you have found them all. Write none if the function $R(x)$ has no points of inflection.

**Solution:** Potential inflection points: Since $R''(x)$ is defined for all $x$, we want all the solutions to $R''(x) = 0$, that is $x = -\frac{10}{11}, x = 1$ and $x = -2$.

Looking at the signs of $R''(x)$ around these points:

$$R''(-3) = (-)(+)(-)=+$$
$$R''(-1) = (-)(+)(+)=−$$
$$R''(0) = (+)(+)(+)=+$$
$$R''(2) = (+)(+)(+)=+$$

OR you can compute the values of $R''(x)$ around these points

$$R''(-3) = 94208$$
$$R''(-1) = -64$$
$$R''(0) = 80$$
$$R''(2) = 2048$$

Since the sign of $R''(x)$ only changes at $x = -2$ and $x = -\frac{10}{11}$ then the inflection points of $R(x)$ are at $x = -2, -\frac{10}{11}$

b. [4 points] Find the quadratic approximation $G(x)$ of $R(x)$ at the point $(-1, 5)$ on the graph of $R(x)$. Show all your work.

**Solution:**

$$G(x) = R(-1) + R'(-1)(x + 1) + \frac{R''(-1)}{2}(x + 1)^2$$

where

$$R(-1) = 5 \quad \text{since } (-1, 5) \text{ is a point on the graph of } R(x).$$

$$R'(-1) = (-1 - 1)^7(-1 + 2)^4 = -128$$

$$R''(-1) = (11(-1) + 10)(-1 - 1)^6(-1 + 2)^3 = -64$$

Hence $G(x) = 5 - 128(x + 1) - 32(x + 1)^2$