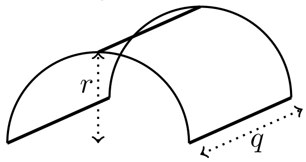
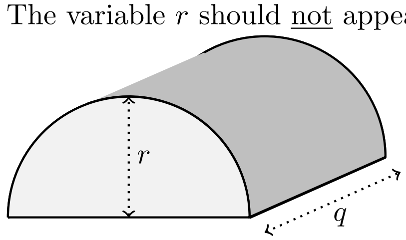
3. [9 points] Duncan's person is making him a new tent in the shape of half a cylinder. She plans to use wire to make the tent frame. This will consist of two semicircles of radius r (measured in inches) attached to three pieces of wire of length q (also measured in inches), as shown in the diagram below. She has 72 inches of wire to use for this.



a. [4 points] Find a formula for r in terms of q.

Solution: The amount of wire S used on the one semicircle of radius r is given by $S = \frac{1}{2}(2\pi r)$ inches. For the rest of the tent, she uses $3q$ inches of wire. Since she used 72 inches of wire to build the tent, we have $r = \dfrac{72-3q}{2\pi}$.

b. [2 points] Let $V(q)$ be the volume (in cubic inches) of the space inside the tent after the fabric is added, given that the total length of wire is 72 inches and the length of the tent is q inches. (Recall that the tent shape is half of a cylinder.) Find a formula for $V(q)$. The variable r should not appear in your answer.



Solution: The volume of enclosed by the tent V is the volume of a half cylinder. In this case $V = \frac{1}{2}\pi r^2 h$, where r is the radius of the semicircular lateral face and h is the length of the tent. In our case $h = q$ and using our answer from part **a** we obtain

$$V = \frac{1}{2}\pi r^2 h = \frac{\pi q}{2}\left(\frac{72-3q}{2\pi}\right)^2$$

c. [3 points] In the context of this problem, what is the domain of $V(q)$?

Solution: The length q of the tent cannot be negative and it has to be smaller than the total amount of wire used 72 inches, then $0 < q < 72$. On the other hand, the more wire you use building the length of the tent q the smallest the radius r will be. If we set our expression for r in terms of q from **a.** to 0: $\dfrac{72-3q}{2\pi} = 0$, we obtain $3q = 72$ and therefore $q = 24$. Hence the domain of $V(q)$ is all values of q that satisfy $0 < q < 24$.