6. [9 points] A group of biology students is studying the length L of a newborn corn snake (in cm) as a function of its weight w (in grams). That is, $L = G(w)$. A table of values of $G(w)$ is shown below.

w	5	10	15	20	25
$G(w)$	24.5	31.6	38.7	44.7	50
$G'(w)$	2.23	1.58	1.30	1.12	1.05

Assume that $G'(w)$ is a differentiable and decreasing function for $0 < w < 25$.

a. [2 points] Find a formula for $H(w)$, the tangent line approximation of $G(w)$ near $w = 20$.

Solution: The formula for is $H(w) = G(20) + G'(20)(w - 20)$. From the table we get $H(w) = 44.7 + 1.12(w - 20)$.

b. [1 point] Use the tangent line approximation of $G(w)$ near $w = 20$ to approximate the length of a corn snake that weighs 22 grams.

Solution: $G(22) \approx H(22) = 1.12(22 - 20) + 44.7 = 46.94$ cm.

c. [2 points] Is your answer in part (b) an overestimate or an underestimate? Circle your answer and write a sentence to justify it.

Solution:

Circle one: $\boxed{\text{Overestimate}}$ Underestimate CANNOT BE DETERMINED

Justification:
Since $G'(w)$ is a differentiable and decreasing function for $0 < w < 25$, then $G(w)$ is concave down on $0 < w < 25$. Hence the values of the tangent line approximation $H(w)$ will be larger than the actual values of $G(w)$ for $0 < w < 25$.

d. [4 points] In their study of the growth of corn snakes, they found the results of a recent article that states that the average weight w of a corn snake (in grams) t weeks after being born is given by $w = \frac{1}{5}t^2$. Let $S(t) = G\left(\frac{1}{5}t^2\right)$ be the length of a corn snake t weeks after being born. Find a formula for $P(t)$, the tangent line approximation of $S(t)$ near $t = 5$.

Solution: The formula for the tangent line approximation $P(t)$ is $P(t) = S(5) + S'(5)(t - 5)$. Since $S(t) = G(\frac{1}{5}t^2)$, then $S'(t) = \frac{2}{5}t \cdot G'(\frac{1}{5}t^2)$. Using these formulas we get that $S(5) = G(\frac{1}{5}(5^2)) = G(5) = 25.4$ and $S'(5) = 2 \cdot G'(5) = 4.46$.

Answer: $P(t) = \underline{24.5 + 4.46(t - 5) = 4.46t + 2.2}$