8. [8 points] At Happy Hives Bee Farm, the population of bees, in thousands, t months after the farm opens, can be modeled by $g(t)$, where

$$g(t) = \begin{cases} 20 + \frac{1}{3}e^{4-t} & \text{for } 0 \le t \le 4 \\ -\frac{1}{6}t^3 + \frac{9}{4}t^2 - 7t + 23 & \text{for } 4 < t \le 8 \end{cases}$$

and

$$g'(t) = \begin{cases} -\frac{1}{3}e^{4-t} & \text{for } 0 < t < 4 \\ -0.5(t-2)(t-7) & \text{for } 4 < t < 8. \end{cases}$$

a. [6 points] Find the values of t that minimize and maximize $g(t)$ on the interval $[0, 8]$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema. For each answer blank, write NONE if appropriate.

Solution: In this case, you need to establish if the function is continuous before using the procedure listed above.

- Continuity of $g(t)$ on $[0, 8]$:
  Since $\lim_{t\to 4^+} g(t) = \lim_{t\to 4^+} -\frac{1}{6}t^3 + \frac{9}{4}t^2 - 7t + 23 = \frac{61}{3}$ and $\lim_{t\to 4^+} g(t) = \frac{61}{3} = \lim_{t\to 4^-} g(t) = g(4)$, $g(t)$ is continuous at 4. Both pieces are continuous, so $g(t)$ is continuous on the interval $[0, 4]$.
- Critical points of $g(t)$: Using the formula for $g'(t)$
  - $g'(t) = 0$ on $(0, 4)$: Since $e^{4-t} \neq 0$ for any value of t, then $g'(t) \neq 0$ on $(0, 4)$.
  - $g'(t) = 0$ on $(4, 8)$: In this case $-0.5(t-2)(t-7) = 0$ if $t = 2, 7$. Hence the only solution in $(4, 8)$ is $t = 7$.
  - $g'(t)$ is undefined on $(0, 8)$. Based on the formula for $g'(t)$, the only point where $g'(t)$ could be undefined is $t = 4$. In this case $g'(4)$ does not exist since:
    - $\lim_{h\to 0^-} \frac{g(4+h) - g(4)}{h} = -\frac{1}{3}e^{4-4} = -\frac{1}{3}$
    - $\lim_{h\to 0^+} \frac{g(4+h) - g(4)}{h} = -0.5(4-2)(4-7) = 3$

  Hence the critical points of $g(t)$ in $(0, 8)$ are $t = 4, 7$.

Next we make a table to list $g(t)$ at all critical points and endpoints, and choose the values of t corresponding to the min and max from the table.

t	0	4	7	8
$g(t)$	38.199	20.33	27.08	25.67

Answer: Global max(es) at $t =$ **0**

Answer: Global min(s) at $t =$ **4**

b. [2 points] What is the largest population of bees that occurs in the first 8 months the farm is open?

Answer: **38.199 thousand (or** $38,199$**)**