3. [12 points] The Public Transit Authorities (PTA) are designing rain shelters for their bus stops. They decide to place a roof in the shape of half a cylinder on four vertical legs of height $y$ feet. The four legs are placed in a rectangle on the ground with width $x$ feet and length $y$ feet.

The costs of production are:

- $\$ 25$ for each foot of the total length of the legs,
- $\$ 40$ for each square foot of the area of the roof.

The following formulas may be useful in this problem:

- the surface area of a cylinder of radius $r$ and length $\ell$ is $2 \pi r \ell$,
- the volume of a cylinder of radius $r$ and length $\ell$ is $\pi r^{2} \ell$.


The PTA would like to spend exactly $\$ 5000$ on one rain shelter.
a. [5 points] Find a formula for $y$ in terms of $x$.

Answer: $y=$
b. [4 points] Find a formula for the total volume in cubic feet covered by the shelter, $V(x)$, if the width of the dashed rectangle has length $x$ feet.

Answer: $V(x)=$ $\qquad$

The statement of the problem has been included for your convenience.
The Public Transit Authorities (PTA) are designing rain shelters for their bus stops. They decide to place a roof in the shape of half a cylinder on four vertical legs of height $y$ feet. The four legs are placed in a rectangle on the ground with width $x$ feet and length $y$ feet.

The costs of production are:

- $\$ 25$ for each 1 foot of the total length of the legs,
- $\$ 40$ for each square foot of the area of the roof.

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- the volume of a cylinder of radius $r$ and length $\ell$ is $\pi r^{2} \ell$.


The PTA would like to spend $\$ 5000$ on one rain shelter.
c. [3 points] The PTA wants to make sure that each of the sides of the rectangle has length at least 5 feet, and the height (that is, $y$ ) of the shelter is at least 8 feet. In the context of the problem, what is the domain of the function $V(x)$ ?

Answer:

