3. [12 points] The Public Transit Authorities (PTA) are designing rain shelters for their bus stops. They decide to place a roof in the shape of half a cylinder on four vertical legs of height $y$ feet. The four legs are placed in a rectangle on the ground with width $x$ feet and length $y$ feet.

The costs of production are:
- $25 for each foot of the total length of the legs,
- $40 for each square foot of the area of the roof.

The following formulas may be useful in this problem:
- the surface area of a cylinder of radius $r$ and length $\ell$ is $2\pi r \ell$,
- the volume of a cylinder of radius $r$ and length $\ell$ is $\pi r^2 \ell$.

The PTA would like to spend exactly $5000 on one rain shelter.

a. [5 points] Find a formula for $y$ in terms of $x$.

\[ y = \]  

Answer: \( y = \)  

b. [4 points] Find a formula for the total volume in cubic feet covered by the shelter, $V(x)$, if the width of the dashed rectangle has length $x$ feet.

\[ V(x) = \]  

Answer: \( V(x) = \)

The problem continues on the next page.
The statement of the problem has been included for your convenience.
The Public Transit Authorities (PTA) are designing rain shelters for their bus stops. They
decide to place a roof in the shape of half a cylinder on four vertical legs of height \( y \) feet. The
four legs are placed in a rectangle on the ground with width \( x \) feet and length \( y \) feet.

The costs of production are:

- $25 for each 1 foot of the total length of the legs,
- $40 for each square foot of the area of the roof.

The following formulas may be useful in this problem:

- the surface area of a cylinder of radius \( r \) and length \( \ell \) is \( 2\pi r\ell \),
- the volume of a cylinder of radius \( r \) and length \( \ell \) is \( \pi r^2\ell \).

The PTA would like to spend $5000 on one rain shelter.

\textbf{c. [3 points]} The PTA wants to make sure that each of the sides of the rectangle has length
at least 5 feet, and the height (that is, \( y \)) of the shelter is at least 8 feet. In the context
of the problem, what is the domain of the function \( V(x) \)?

Answer: ______________________