3. [12 points] The Public Transit Authorities (PTA) are designing rain shelters for their bus stops. They decide to place a roof in the shape of half a cylinder on four vertical legs of height y feet. The four legs are placed in a *rectangle* on the ground with width x feet and length y feet.

The costs of production are:

- \$25 for each foot of the total length of the legs,
- \$40 for each square foot of the area of the roof.

The following formulas may be useful in this problem:

- the surface area of a cylinder of radius r and length ℓ is $2\pi r\ell$,
- the volume of a cylinder of radius r and length ℓ is $\pi r^2 \ell$.



The PTA would like to spend exactly \$5000 on one rain shelter.

a. [5 points] Find a formula for y in terms of x.

Answer: y =_____

b. [4 points] Find a formula for the total volume in cubic feet covered by the shelter, V(x), if the width of the dashed rectangle has length x feet.

Answer: V(x) =_

The problem continues on the next page.

The statement of the problem has been included for your convenience.

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The PTA would like to spend \$5000 on one rain shelter.

c. [3 points] The PTA wants to make sure that *each* of the sides of the rectangle has length at least 5 feet, and the height (that is, y) of the shelter is at least 8 feet. In the context of the problem, what is the domain of the function V(x)?