**1**. [13 points] Some values of the twice differentiable function $f(x)$ and of its first and second derivative are given by the following table

| $x$ | 0 | 1 | 2 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | | | 4 | 4.3 | 5 | |
| $f'(x)$ | | | 8 | | 0.25 | 0.6 | 2 |
| $f''(x)$ | 4 | | | | 0.1 | 0.2 | |

Suppose the function $f(x)$ is defined and invertible for $-\infty < x < \infty$. In the following questions, you will find some of the missing values using the information given. If there is not enough information given to answer the question, write "NEI". Show your work.

**a**. [4 points] The function $a(x) = \ln(1 + f(x))$ satisfies $a'(2) = 2$. Find $f(2)$.

*Solution:*

$$a'(x) = \frac{1}{1+f(x)} f'(x)$$
$$2 = \frac{8}{1+f(2)}$$
$$8 = 2 + 2f(2)$$
$$f(2) = 3$$

**Answer:** $f(2) = 3$.

**b**. [3 points] Let $b(x) = f(x)f'(x)$ and $b'(0) = 4$. Find $f'(0)$.

*Solution:*

$$b'(x) = (f'(x))^2 + f(x)f''(x)$$
$$4 = (f'(0))^2 + f(0)f''(0)$$
$$4 = (f'(0))^2 + 4$$
$$f'(0) = 0.$$

**Answers:** $f'(0) = 0$.

**c**. [3 points] The quadratic approximation $Q(x)$ of the function $f(x)$ at $x = 1$ is $Q(x) = \frac{1}{2}x + \frac{3}{2}$. Find $f(1)$, $f'(1)$, and $f''(1)$.

*Solution:*

**Answers:** $f(1) = 2$ , $f'(1) = \frac{1}{2}$ , $f''(1) = 0$

**d**. [3 points] Let $h(x) = f^{-1}(x)$. Find the value of $h'(5)$.

*Solution:* $h'(x) = \frac{1}{f'(f^{-1}(x))}$, then $h'(5) = \dfrac{1}{f'(f^{-1}(5))} = \dfrac{1}{f'(6)} = \dfrac{1}{0.6} = \dfrac{5}{3}$.

**Answer:** $h'(5) = \dfrac{5}{3}$