4. [12 points] In the following questions, use calculus to justify your answers and show enough evidence to demonstrate that you have found them all. Determine your answers algebraically.

a. [7 points] Let $f(x)$ be a continuous function defined for all real numbers with derivative given by

$$f'(x) = \frac{(2x+1)(x-2)^2}{(x+3)^{\frac{1}{3}}}.$$

Find the x-coordinate(s) of the local maximum(s) and local minimum(s) of the function $f(x)$. Write "NONE" if the function has no local maximum(s) and/or local minimum(s).

Solution: The critical points of $f(x)$ are $x = -3$, $-\frac{1}{2}$, and 2.
To classify them, note that

- for $x < -3$, $f'(x) = \frac{(-)(+)}{(-)} = (+)$,
- for $-3 < x < -\frac{1}{2}$, $f'(x) = \frac{(-)(+)}{(+)} = (-)$,
- for $-\frac{1}{2} < x < 2$, $f'(x) = \frac{(+)(+)}{(+)} = (+)$,
- for $x > 2$, $f'(x) = \frac{(+)(+)}{(+)} = (+)$.

Therefore, $f(x)$ has a local maximum at $x = -3$ and a local minimum at $x = -\frac{1}{2}$.

Answers: Local maximum(s) at $x = -3$ Local minimum(s) at $x = -\frac{1}{2}$

b. [5 points] Let $g(x)$ be a continuous function defined for all real numbers with second derivative given by

$$g''(x) = (2^x - 4)(x^2 - 4).$$

Find the x-coordinates of the inflection points of the function $g(x)$. Write "NONE" if the function has no inflection points.

Solution: Note that $g''(x)$ is defined everywhere, and $g''(x) = 0$ for $x = -2$ and $x = 2$. We check for a change of sign in $g''(x)$ at these points to see if they are inflection points:

- for $x < -2$, $g''(x) = (-)(+) = (-)$,
- for $-2 < x < 2$, $g''(x) = (-)(-) = (+)$,
- for $x > 2$, $g''(x) = (+)(+) = (+)$.

Therefore, $x = -2$ is the only inflection point of $g(x)$.

Answer: Inflection point(s) at $x = -2$