5. [15 points] The graph of the function $f(x)$ with domain $-4 \leq x \leq 8$ is shown below.

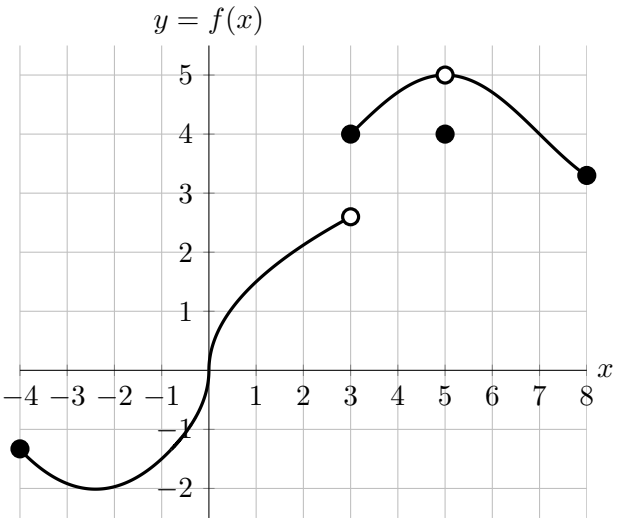
The function $f(x)$ satisifies:

- $f(x) = 1.5x^{\frac{1}{3}}$

  for $-1 < x < 1$,

- $f(x) = 4 + \sin\left(\frac{\pi}{4}(x-3)\right)$

  for $3 \leq x < 5$ and $5 < x \leq 8$.



a. [2 points] Estimate the x-coordinate(s) of all the local minimum(s) of $f(x)$ in $-4 < x < 8$. Write "NONE" if $f(x)$ does not have any local minimums.

Solution:

Answer: $x = -2.4, 5$

b. [3 points] Find the value(s) of b in $-4 < b < 8$ for which the limit $\lim_{h \to 0} \dfrac{f(b+h) - f(b)}{h}$ does *not exist*. Write "NONE" if there are no such values of b.

Solution:

Answer: $b = 0, 3, 5$

c. [4 points] Estimate the x-coordinate(s) of all critical points of $f(x)$ in $-4 < x < 8$. Write "NONE" if $f(x)$ does not have any critical points.

Solution:

Answer: $x = -2.4, 0, 3, 5$

d. [3 points] On which of the following intervals is the *conclusion* of the Mean Value Theorem true? Circle your answer.

Solution:

$[-4, 0]$ $[0, 5]$ $[1, 3]$ $[3, 7]$ NONE

e. [3 points] On which of the following intervals are the *hypotheses* of the Mean Value Theorem true? Circle your answer.

Solution:

$[-3, -1]$ $[-2, 2]$ $[0, 2]$ $[3, 5]$ NONE