6. [5 points] The function $P(t)$ is given by the equation

$$P(t) = \begin{cases} t+4 & t<2 \\ t^2-3t+8 & 2 \le t \le 3 \\ \frac{1}{9}(t^3+44) & t>3 \end{cases}$$

For which values of t is $P(t)$ *differentiable*? Show all your work to justify your answer.

Solution:

- At $t=2$:
  - Continuity:

$$\lim_{t\to 2^-} t+4 = 6 \quad \text{and} \quad \lim_{t\to 2^+} t^2-3t+8 = 6 \quad \text{and} \quad P(2)=6.$$

    Hence $P(t)$ is continuous at $t=2$.
  - Differentiability: The function $g(t)=t+4$ satisfies $g'(t)=1$ and hence $g'(2)=1$. Similarly, it $h(t)=t^2-3t+8$ satisfies $h'(t)=2t-3$ and $h'(2)=1$. So

$$\lim_{t\to 2-} P'(t) = 1 = \lim_{t\to 2+} P'(t).$$

  Hence $P(t)$ is differentiable at $t=2$ (with $P'(2)=1$).
- At $t=3$:
  - Continuity:

$$\lim_{t\to 3^-} t^2-3t+8 = 8 \quad \text{and} \quad \lim_{t\to 3^+} \frac{1}{9}(t^3+44) = \frac{71}{9} \neq 8.$$

    Hence $P(t)$ is not continuous at $t=3$ and therefore not differentiable at $t=3$.
- All the functions (polynomials) involved in the formula of $P(t)$ are differentiable on the domains assigned to them.

Hence $P(t)$ is differentiable for all $t \neq 3$.

Answer: The function $P(t)$ is differentiable for the following values of t: all $t \neq 3$.