7. [10 points] The amount of chlorine in a chemical reaction $C(t)$ (in gallons) t seconds after it has been added into a solution is given by the function

$$C(t) = 2 - 3(t-5)^{\frac{4}{5}}(t-1)e^{-t} \qquad \text{for} \qquad t \geq 0.$$

Notice that

$$C'(t) = \frac{3(t-6)(5t-9)e^{-t}}{5(t-5)^{1/5}}.$$

a. [8 points] Use calculus to find the time(s) (if any) at which the amount of chlorine in the solution is the greatest and the smallest. If the function has no global maximum or global minimum write "NONE" in the appropriate space. Show all your work.

Solution: Critical points:

- $C'(t) = 0$: $t = 6$ and $t = 1.8$.
- $C'(t)$ undefined: $t = 5$.

Finding the output values of $C(t)$ at critical points and the behavior of the function at the endpoints:

t	0	1.8	5	6
$C(t)$	$2 + 3(5)^{\frac{4}{5}} \approx 12.87$	≈ 0.994	2	≈ 1.96

$$\lim_{t\to\infty} 2 - 3(t-5)^{\frac{4}{5}}(t-1)e^{-t} = 2.$$

Answer: Global maximum(s) at $t = 0$

Global minimum(s) at $t = 1.8$.

b. [2 points] What is the maximum amount of chlorine in the solution? If there is never a maximum amount of chlorine in the solution, write "NONE".

Solution:

Answer: $2 + 3(5)^{\frac{4}{5}} \approx 12.87$ gallons.