8. [8 points]

Kristen is machining a metal washer to fix her broken down motorcycle. A washer is a flat, circular piece of metal with a hole in the middle. Kristen's washer is depicted by the shaded region in the figure to the right. The washer has an inner radius of r centimeters and an outer radius of R centimeters. The area of the washer must be exactly 5 square centimeters, and r must be at least 1 centimeter.
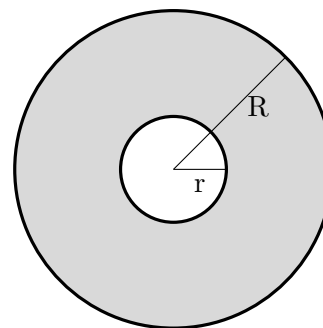
a. [3 points] Find a formula for r in terms of R.

Answer: $r =$ _____

b. [2 points] The structural integrity of the washer depends on both its inner radius and its outer radius. Specifically, the structural integrity is given by the equation

$$S = 32R\left(\ln(rR+1) + 7\right).$$

Express S as a function of R. *Your answer should not include r.*

Answer: $S(R) =$ _____

c. [3 points] What is the domain of $S(R)$ in the context of this problem? You may give your answer as an interval or using inequalities.

Answer: _____