

10. [8 points] Let $j(t)$ be a differentiable function with domain $(0, \infty)$ that satisfies all of the following:

- $j(5) = 0$
- $j(t)$ has exactly two critical points
- $j(t)$ has a local maximum at $t = 5$
- $j(t)$ has a local minimum at $t = 9$
- $\lim_{t \rightarrow 0^+} j(t) = -\infty$
- $\lim_{t \rightarrow \infty} j(t) = 0$

You do not need to show work in this problem.

a. [2 points] Circle all of the following intervals on which $j'(t)$ must always be negative.

(0, 2) (2, 5) (5, 9) (9, ∞)

b. [3 points] Find all the values of t at which $j(t)$ attains global extrema on the interval $[1, 9]$. If not enough information is provided, write NOT ENOUGH INFO. If there are no such values of t , write NONE.

Solution: We can conclude that $j(t)$ increases until $t = 5$, then decreases until $t = 9$. So, $t = 5$ must be the global max.

By the Extreme Value Theorem, we know $j(t)$ has a global minimum on $[1, 9]$, but we don't know which value, $j(1)$ or $j(9)$, is smaller.

Answer: Global max(es) at $t =$ 5

Answer: Global min(s) at $t =$ NOT ENOUGH INFO

c. [3 points] Find all the values of t at which $j(t)$ attains global extrema on its domain. If not enough information is provided, write NOT ENOUGH INFO. If there are no such values of t , write NONE.

Solution: Similarly to part b., we can conclude that $j(t)$ increases until $t = 5$, then decreases until $t = 9$, then increases again. Since $j(5) = 0$, we know $t = 5$ must be the global max, since $j(t)$ can never again reach 0. (If it did, there would be another critical point, since $\lim_{t \rightarrow \infty} j(t) = 0$.)

Also, we know there is no global min since $\lim_{t \rightarrow 0^+} j(t) = -\infty$.

Answer: Global max(es) at $t =$ 5

Answer: Global min(s) at $t =$ NONE