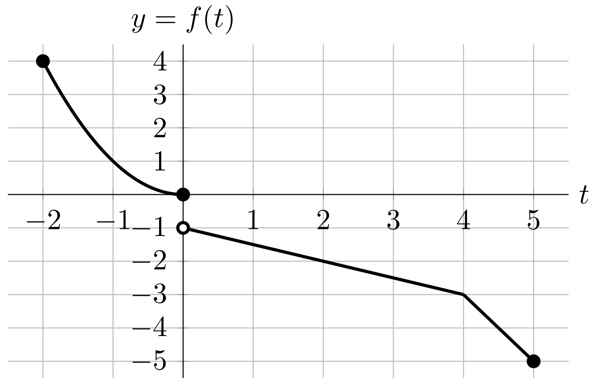
2. [16 points]

Shown to the right is the graph of a function $f(t)$.

Note that:

- $f(t) = t^2$ on $[-2, 0]$,
- $f(t)$ is linear on the intervals (0,4) and (4,5).



a. Evaluate each of the following quantities **exactly**, or write DNE if the value does not exist. You do not need to show work, but limited partial credit may be awarded for work shown.

i. [2 points] Find $(f^{-1})'(-2)$.

Solution: $(f^{-1})'(-2) = \dfrac{1}{f'(f^{-1}(-2))} = \dfrac{1}{f'(2)} = \dfrac{1}{-1/2} = -2$

Answer: $(f^{-1})'(-2) =$ -2

ii. [2 points] Let $g(t) = \sin(t)f(t)$. Find $g'(4)$.

Solution: We have $g'(t) = \sin(t)f'(t) + \cos(t)f(t)$, so $g'(4) = \sin(4)f'(4) + \cos(4)f(4)$. Since $f'(4)$ DNE, $g'(4)$ DNE. (This is because, if $g'(4)$ existed, then since $f(t) = g(t)/\sin(t)$, we would have that $f'(4) = (\sin(4)g'(4) - g(4)\cos(4))/\sin^2(4)$ also existed.)

Answer: $g'(4) =$ **DNE**

iii. [4 points] Let $h(t) = \dfrac{f(2t+2)}{2^t}$. Find $h'(0)$.

Solution: We have $h'(t) = \dfrac{2^t f'(2t+2)\cdot 2 - f(2t+2)\ln(2)2^t}{(2^t)^2}$, so

$$h'(0) = \frac{2^0 f'(2)\cdot 2 - f(2)\ln(2)2^0}{(2^0)^2} = 1(-1/2)(2) - (-2)\ln(2) = -1 + 2\ln(2).$$

Answer: $h'(0) =$ $-1 + 2\ln(2)$

iv. [4 points] Let $j(t) = \ln(-f'(t))$. Find $j'(-1)$.

Solution: We have $j'(t) = \dfrac{1}{-f'(t)}(-f''(t))$.

Since $[t^2]' = 2t$ and $[t^2]'' = 2$, we have $f'(-1) = -2$ and $f''(-1) = 2$, so

$$j'(-1) = \frac{1}{-f'(-1)}(-f''(-1)) = \frac{1}{-(-2)}(-2) = -1.$$

Answer: $j'(-1) =$ **-1**

b. [2 points] On which of the following interval(s) does $f(t)$ satisfy the hypotheses of the Mean Value Theorem? Circle all correct choices.

$[-2, 5]$ [0,3]   [3,5] $\boxed{\text{NONE OF THESE}}$

c. [2 points] On which of the following interval(s) does $f(t)$ satisfy the conclusion of the Mean Value Theorem? Circle all correct choices.

$\boxed{[-2, 5]}$ $[0, 3]$ $[3, 5]$ NONE OF THESE