3. [11 points] Suppose $h(x)$ is a continuous function defined for all real numbers x. The derivative and second derivative of $h(x)$ are given by

$$h'(x) = \frac{2x}{3(x^2-1)^{2/3}} \qquad \text{and} \qquad h''(x) = -\frac{2(x^2+3)}{9(x^2-1)^{5/3}}.$$

a. [6 points] Find the x-coordinates of all critical points of $h(x)$ and all values of x at which $h(x)$ has a local extremum. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank below, write NONE if appropriate.

Solution: The critical points occur at values of x where $h'(x) = 0$ or $h'(x)$ does not exist. We have $h'(x) = 0$ at $x = 0$, and $h'(x)$ does not exist at -1 and 1. Since h is continuous for all real numbers, these three values are all in its domain and so all are critical points.

Now we need to know the sign of $h'(x)$ on four intervals. To determine this, we consider the signs of the numerator and denominator.

	sign of $2x$	sign of $3(x^2-1)^{2/3}$	sign of $h'(x)$
$-\infty < x < -1$	$-$	$+$	$-$
$-1 < x < 0$	$-$	$+$	$-$
$0 < x < 1$	$+$	$+$	$+$
$1 < x < \infty$	$+$	$+$	$+$

Thus $h'(x)$ is decreasing for $x < 0$ and increasing for $x > 0$, so we have a local minimum at $x = 0$ and no local maxima.

Answer: Critical point(s) at $x =$ **-1,0,1**

Answer: Local max(es) at $x =$ **NONE**

Answer: Local min(s) at $x =$ **0**

b. [5 points] Find the x-coordinates of all inflection points of $h(x)$. If there are none, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

Solution: Inflection points can only occur at values of x where $h''(x) = 0$ or $h''(x)$ does not exist. There are no places where $h''(x) =)$, and $h''(x)$ does not exist at -1 and 1. Since h is continuous for all real numbers, these three values are all candidates for inflection points.

Now we need to know the sign of $h''(x)$ on three intervals. To determine this, we consider the signs of the numerator and denominator, plus we take into account the negative sign in front.

	$-$ sign in front	sign of $2(x^2+3)$	sign of $9(x^2-1)^{5/3}$	sign of $h'(x)$
$-\infty < x < -1$	$-$	$+$	$+$	$-$
$-1 < x < 1$	$-$	$+$	$-$	$+$
$1 < x < \infty$	$-$	$+$	$+$	$-$

Thus $h''(x)$ changes sign at -1 and 1, so $h(x)$ changes concavity at both of these points.

Answer: Inflection point(s) at $x =$ **-1,1**