4. [10 points]

a. Let $\mathcal{C}$ be the curve given by the equation

$$y\cos(2x) = y^3 + b,$$

where b is a constant. The curve $\mathcal{C}$ passes through the point $(0, 2)$.

i. [2 points] Find b.

Solution: Plugging in $(0, 2)$, we find that

$$2\cos(2\cdot 0) = 2^3 + b$$
$$2 = 8 + b$$
$$b = -6.$$

Answer: $b =$ **-6**

ii. [5 points] For the curve $\mathcal{C}$, find a formula for $\frac{dy}{dx}$ in terms of x and y. To earn credit for this problem, you must compute this by hand and show every step of your work clearly.

Solution:
Using implicit differentiation, and the product rule on the left-hand side,

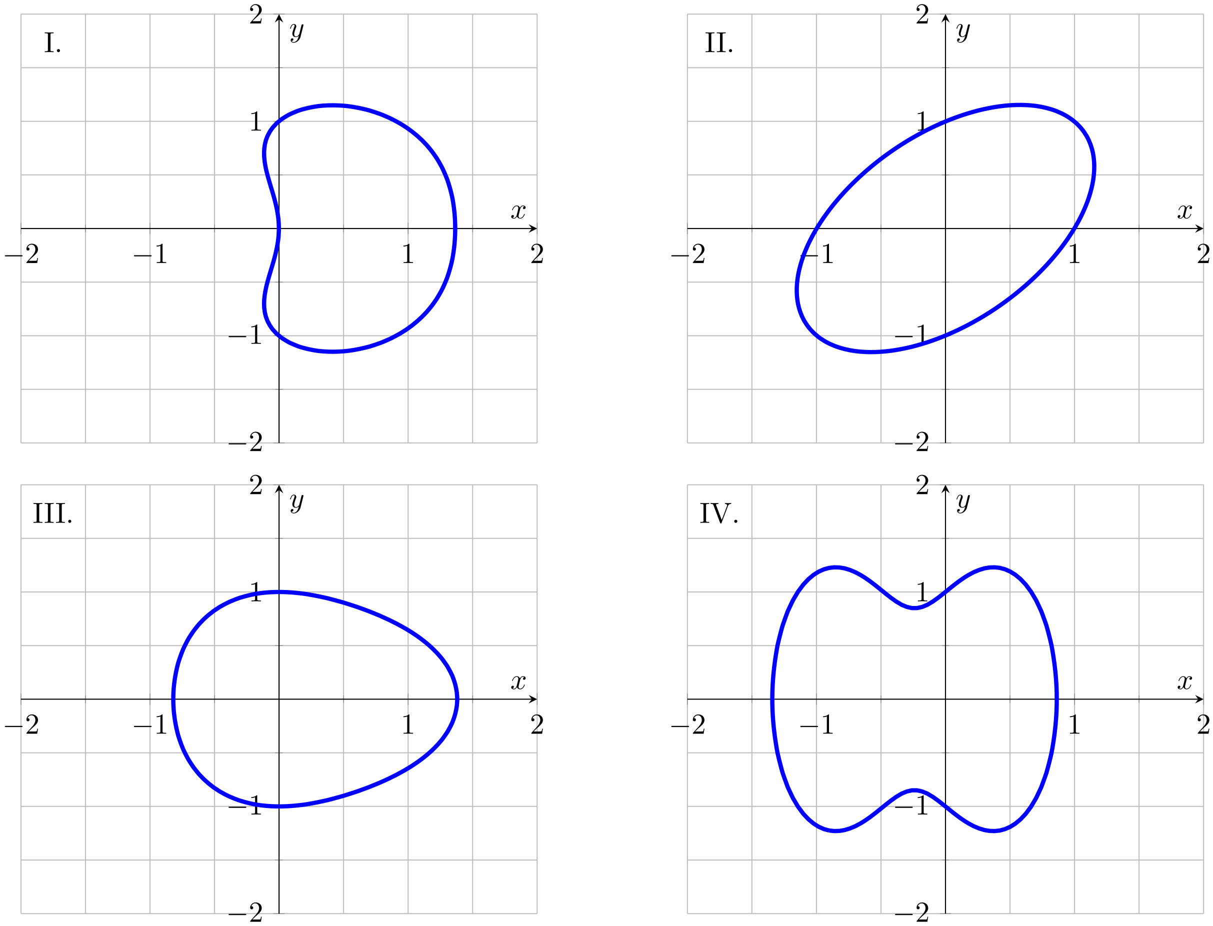
$$-y\sin(2x)\cdot 2 + \frac{dy}{dx}\cos(2x) = 3y^2\frac{dy}{dx}$$
$$\frac{dy}{dx}\cos(2x) - 3y^2\frac{dy}{dx} = 2y\sin(2x)$$
$$\frac{dy}{dx}\left(\cos(2x) - 3y^2\right) = 2y\sin(2x)$$
$$\frac{dy}{dx} = \frac{2y\sin(2x)}{\cos(2x) - 3y^2}$$

Answer: $\frac{dy}{dx} =$ $\frac{dy}{dx} = \frac{2y\sin(2x)}{\cos(2x) - 3y^2}$

b. [3 points] A different curve $\mathcal{R}$ passes through the point $(0, 1)$ and satisfies

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}.$$

One of the following graphs is the graph of $\mathcal{R}$. Which of the graphs is it? Write the numeral (I, II, III, or IV) of the graph you choose on the answer line at the bottom of this page.



Solution: We find that the slope at the given point $(0, 1)$ is $1/2$, so this rules out III. Finding that the slope at the point $(0, -1)$ must also be $1/2$, we conclude that II must be correct. (We could also have ruled out I and IV (and III) by noting that these graphs have vertical tangents when $y = 0$, but dy/dx is not undefined when $y = 0$.)

Answer: **II**