6. [10 points]

a. [4 points] Below is a table of values for a differentiable function $g(x)$. Also shown are some values of $g'(x)$, which is an increasing function and also differentiable.

x	3	8	10
$g(x)$	10	1	0
$g'(x)$	-4	0.6	2

i. [2 points] Write a formula for $L(x)$, the linear approximation of $g(x)$ at $x = 3$.

Solution: The general formula for the linear approximation is $L(x) = g(a) + g'(a)(x - a)$. In our case, $a = 3$, $g(3) = 10$, and $g'(3) = -4$. So $L(x) = 10 - 4(x - 3)$.

Answer: $L(x) =$ $10 - 4(x - 3)$

ii. [1 point] Use your formula for $L(x)$ to estimate $g(3.2)$.

Solution: $g(3.2) \approx L(3.2) = 10 - 4(3.2 - 3) = 10 - 4(.2) = 9.2$

Answer: $g(3.2) \approx$ 9.2

iii.[1 point] Is your estimate of $g(3.2)$ an overestimate or an underestimate? Circle your answer.

Solution: Since $g'(x)$ is an increasing function,$g''(x)$ is positive, and $g(x)$ is concave up. The linear approximation of a concave up function is always an underestimate.

Overestimate [✓ *Underestimate*] Cannot be determined

b. [2 points] The quadratic approximation of $g(x)$ at $x = 10$ is

$$Q(x) = 2(x - 10) + 2(x - 10)^2.$$

Find $g''(10)$.

Solution: Since we know $Q(x) = g(10) + g'(10)(x - 10) + \frac{g''(10)}{2}(x - 10)^2$, we must have $\frac{g''(10)}{2} = 2$, and $g''(10) = 4$.

Answer: $g''(10) =$ **4**

c. [4 points] Let $h(x) = (g(x))^3$. The linear approximation of $h(x)$ at $x = 6$ is

$$K(x) = 8 + 3(x - 6).$$

Find $g(6)$ and $g'(6)$.

Solution: We know $K(x) = h(6) + h'(6)(x - 6)$, so $h(6) = (g(6))^3 = 8$, and $g(6) = 2$. Now we find $g'(6)$. By the chain rule we have $h'(x) = 3(g(x))^2 g'(x)$. Since $h'(6) = 3$, we have $3(g(6))^2 g'(6) = 3$, or $3(2)^2 g'(6) = 3$. That is, $g'(6) = 1/4$.

Answer: $g(6) =$ **2**

Answer: $g'(6) =$ **1/4**