9. [9 points] Consider the function

$$f(x) = \begin{cases} -2e^{2x-2} & x \leq 1 \\ x^3 - 3x^2 & x > 1. \end{cases}$$

a. [5 points] Find all critical point(s) of $f(x)$. Write NONE if there are none.

Solution: The derivative of $y = -2e^{2x-2}$ is $\frac{dy}{dx} = -4e^{2x-2}$. So there are no critical points for $x < 1$. Also, at $x = 1$, this piece has a slope of -4.

The derivative of $y = x^3 - 3x^2$ is $\frac{dy}{dx} = 3x^2 - 6x = 3x(x-2)$ which is zero at $x = 0$ and $x = 2$. Only $x = 2$ is on the domain of this piece, so $x = 2$ is a critical point and $x = 0$ is not. Also, at $x = 1$, this piece has a slope of -3.

Since the slopes aren't equal on the left and right sides of $x = 1$, the function $f(x)$ can't be differentiable there. So $x = 1$ is also a critical point.

Answer: Critical point(s) at $x =$ **1 and 2**

b. [4 points] Find the x-coordinate of all global maxima and global minima of $f(x)$ on the interval $(-\infty, 4]$. For each, write NONE if there are none.

Solution: Since $f(x)$ is continuous (because both pieces equal -2 when $x = 1$), we can compare the value of $f(x)$ at the critical points $x = 1$ and $x = 2$ along with the end point $x = 4$, and we need to consider the end behavior as $x \to \infty$:

$f(1) = -2e^{2(1)-2} = -2e^0 = -2$

$f(2) = (2)^3 - 3(2)^2 = 8 - 12 = -4$

$f(4) = (4)^3 - 3(4)^2 = 64 - 48 = 12$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} -2e^{2x-2} = 0$

So, $f(x)$ has a global maximum at $x = 4$ and a global minimum at $x = 2$.

Answer: global max(es) at $x =$ 4

Answer: global min(s) at $x =$ 2