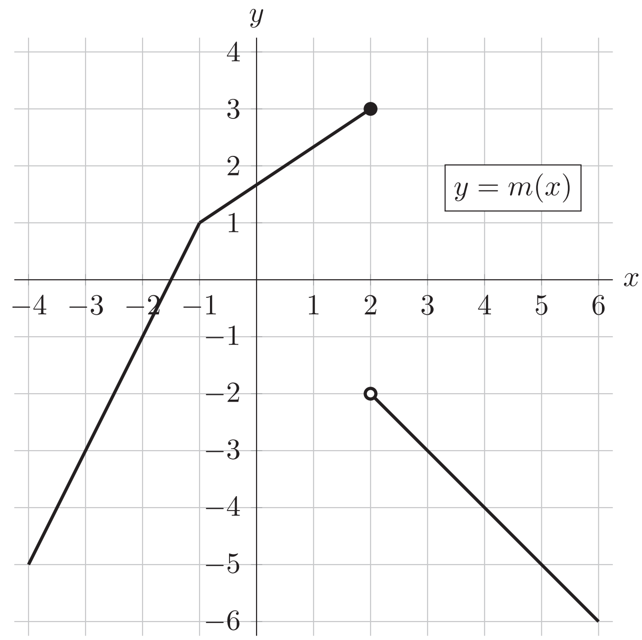
Note: exam problem numbering is off by 1

3. [11 points]

The function $m(x)$ is defined on $(-\infty, \infty)$.

A portion of the graph of the function $m(x)$ is shown.

Note that $m(x)$ is linear on the intervals $(-4,-1)$, $(-1,2)$ and $(2,6)$.



a. [8 points] Evaluate each of the following quantities **exactly**, or write DNE if the value does not exist. You do not need to show work, but limited partial credit may be awarded for work shown. Your answers should not contain the letter m, but do not need to be fully simplified.

i. [2 points] Let $v(x) = x^3 m(x)$. Compute $v'(4)$.

Solution: $v'(x) = 3x^2 m(x) + x^3 m'(x)$. Thus $v'(4) = 48 * (-4) + 64(-1) = -256$.

Answer: $v'(4) =$ -256

ii. [2 points] Let $u(x) = 5m(x-1) + 8$. Compute $u'(3)$.

Solution: $u(x)$ is not continuous at $x = 3$.

Answer: $u'(3) =$ **DNE**

iii. [2 points] Let $w(x) = \dfrac{1}{m(x)}$. Compute $w'(-3)$.

Solution: $w'(x) = -\dfrac{m'(x)}{m(x)^2}$. Thus $w'(-3) = -\frac{2}{9}$.

Answer: $w'(-3) =$ $-\frac{2}{9}$

iv. [2 points] Let $r(x) = \sin(\pi m(x))$. Compute $r'(-2)$.

Solution: $r'(x) = \pi m'(x)\cos(\pi m(x))$. Thus $r'(-2) = 2\pi\cos(-\pi) = -2\pi$.

Answer: $r'(-2) =$ -2π

b. [3 points] Suppose $j(x)$ is a function whose **derivative** is given by the above graph (i.e. $j'(x) = m(x)$). Find a formula for $Q(x)$, the quadratic approximation of $j(x)$ at $x = 5$, assuming $j(5) = 4$.

Answer: $Q(x) =$ $-\frac{1}{2}(x-5)^2 - 5(x-5) + 4$