Note: exam problem numbering is off by 1

4. [12 points]
Suppose $h(x)$ is a continuous function defined for all real numbers x. The **derivative** and **second derivative** of $h(x)$ are given by

$$h'(x) = (x-13)^2(x+4)^{3/7} \quad \text{and} \quad h''(x) = \frac{17(x-13)(x+1)}{7(x+4)^{4/7}}.$$

a. [6 points] Find the x-coordinates of all local extrema of $h(x)$. If there are none of a particular type, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

Solution: The critical points of $h(x)$ are at $x = 13, -4$. Applying the first derivative test we have:

	$x < -4$	$-4 < x < 13$	$x > 13$
$h'(x)$	$+ \cdot - = -$	$+ \cdot + = +$	$+ \cdot + = +$

Answer: Local max(es) at x = **None** Local min(s) at x = -4

b. [6 points] Find the x-coordinates of all inflection points of $h(x)$. If there are none, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

Solution: The second derivative is zero at $x = 13, -1$ and undefined at $x = -4$. We need to check if the sign of $h''(x)$ changes at these points.

	$x < -4$	$-4 < x < -1$	$-1 < x < 13$	$x > 13$
$h''(x)$	$\frac{-\cdot-}{+} = +$	$\frac{-\cdot-}{+} = +$	$\frac{-\cdot+}{+} = -$	$\frac{+\cdot+}{+} = +$

Answer: Inflection Point(s) at x = $-1, 13$