

Note: exam problem numbering is off by 1

5. [12 points] Isabelle is a bee keeper who wants to sell honey at the local farmers market. Let $y = H(d)$ be the amount of honey, in pounds, that Isabelle will sell in a month if she charges d dollars per pound of honey. The functions $H(d)$ and $H'(d)$ are defined and differentiable for all $d \geq 0$. Some values are given in the table below.

d	5.00	5.75	6.50	7.25	8.00	8.75
$H(d)$	59	52	46	38	29	23
$H'(d)$	-10.4	-9.1	-7.8	-11.0	-12.2	-7.6

Assume that $H(d)$ is decreasing and that between each pair of consecutive values of d given in the table, $H'(d)$ is either always increasing or always decreasing.

- a. [3 points] Write a formula for the linear approximation $L(d)$ of $H(d)$ near $d = 6.50$, and use it to estimate the amount of honey, in pounds, Isabelle will sell if she charges \$6.30 per pound.

Answer: $L(d) = \underline{\hspace{10em} -7.8(d - 6.5) + 46 \hspace{10em}}$

Answer: $\approx \underline{\hspace{10em} \$47.56 \hspace{10em}}$

- b. [2 points] Is your estimate from the previous part an overestimate, an underestimate, neither, or is there not enough information to decide? Briefly explain your answer.

Solution: At $d = 6.3$, $H'(d)$ is increasing. Thus $H(d)$ is concave up, and so the linear approximation is an underestimate.

- c. [3 points] Write a formula for the linear approximation $K(y)$ of $(H^{-1})(y)$ near $y = 31$.

Solution: Not enough information due to a typo; all students earned these 3 points on the exam.

- d. [2 points] Use the table to approximate $H''(8.75)$.

Answer: $H''(8.75) \approx \underline{\hspace{10em} \frac{H'(8.75) - H'(8)}{8.75 - 8} \approx 6.133 \hspace{10em}}$

- e. [2 points] The hypotheses of the Mean Value Theorem are satisfied for $H(d)$ on the interval $[5.00, 5.75]$. The conclusion of the theorem then tells you that there is a c in the interval $[5, 5.75]$ so that

$\underline{\hspace{2em} H'(c) \hspace{2em}} = \underline{\hspace{2em} \frac{H(5.75) - H(5)}{5.75 - 5} \approx -9.33 \hspace{2em}}$