

**Note: exam problem numbering is off by 1**

6. [10 points] Let  $P = F(t)$  be the size, in thousands of people, of a certain band's fan club  $t$  years after the beginning of 2020. Formulas modeling  $F(t)$  and  $F'(t)$ , the **derivative** of  $F(t)$ , are given below.

$$F(t) = 175 + 35(t^3 - 7t^2 + 13t - 5)e^{-t} \quad \text{and} \quad F'(t) = -35(t-1)(t-3)(t-6)e^{-t}.$$

In both parts below, you must use calculus to find your answers, and be sure to show enough evidence to fully justify your answers. For each answer blank, write NONE if appropriate.

- a. [5 points] During the first two years after the beginning of 2020 (i.e. for  $0 \leq t \leq 2$ ), when will the band's fan club have the largest and the smallest number of members?

*Solution:* The only critical point in the domain is  $t = 1$ . Testing the value of  $F(t)$  at the critical point and endpoints, we have

$$F(0) = 0$$

$$F(1) = 175 + \frac{70}{e} \approx 200.75$$

$$F(2) = 175 + \frac{35}{e^2} \approx 179.74$$

**Answer:** Largest at  $t =$  1 Smallest at  $t =$  0

- b. [5 points] After the beginning of 2022 (i.e. for  $t \geq 2$ ), what are the largest and smallest number of members the band's fan club will have?

*Solution:* The critical points in the domain are  $t = 3$  and  $t = 6$ . Testing the value of  $F(t)$  at the critical points and determining the end behavior, we have

$$F(2) = 175 + \frac{35}{e^2} \approx 179.74$$

$$F(3) = 175 - \frac{70}{e^3} \approx 171.51$$

$$F(6) = 175 + \frac{1295}{e^6} \approx 178.21$$

$$\lim_{t \rightarrow \infty} F(t) = 175$$

**Answer:** Largest number of members, in thousands:

179.74

**Answer:** Smallest number of members, in thousands:

171.51