10. [10 points] Consider a continuous function $f(x)$, and suppose that $f(x)$ and its first derivative $f'(x)$ are differentiable everywhere. Suppose we know the following information about $f(x)$ and its first and second derivatives.

- On the interval $(-\infty, -2)$, we have $f(x) = 2^{-x}$.
- $\lim_{x\to\infty} f(x) = 6$.
- $f(2) = -5$, $f(3) = 7$, and $f(4) = 8$.
- $f'(x)$ is equal to 0 at $x = -1$, 2, 4, and not at any other x-values.
- $f''(x) < 0$ on the intervals $-1 < x < 0$ and $3 < x < 5$, and not on any other interval.

For each part below, you must use calculus to find and justify your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

a. [5 points] Find the x-coordinates of

i. the global minimum(s) of $f(x)$ on $[3, \infty)$ and

ii. the global maximum(s) of $f(x)$ on $[3, \infty)$.

If there are none of a particular type, write NONE. If there is not enough information to find a desired x-coordinate, write NEI.

b. [5 points] Find the x-coordinates of

i. the global minimum(s) of $f(x)$ on $(-\infty, \infty)$ and

ii. the global maximum(s) of $f(x)$ on $(-\infty, \infty)$.

If there are none of a particular type, write NONE. If there is not enough information to find a desired x-coordinate, write NEI.