8. [11 points] Suppose \( J(x) \) is a continuous function defined for all real numbers \( x \). The derivative and second derivative of \( J(x) \) are given by
\[
J'(x) = \frac{x^2(x - 1)}{\sqrt[3]{x + 4}} \quad \text{and} \quad J''(x) = \frac{x(8x^2 + 31x - 24)}{3(\sqrt[3]{x + 4})^4}.
\]

a. [2 points] Find the \( x \)-coordinates of all critical points of \( J(x) \). If there are none, write NONE.

Throughout parts b. and c. below, you must use calculus to find and justify your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

b. [5 points] Find the \( x \)-coordinates of
i. all local minima of \( J(x) \) and
ii. all local maxima of \( J(x) \).

If there are none of a particular type, write NONE.

c. [4 points] The polynomial \( 8x^2 + 31x - 24 \) (from the numerator of \( J''(x) \)) has two zeroes \( a \) and \( b \), where \( a \approx -4.54 \) and \( b \approx 0.66 \). How many inflection points does the function \( J(x) \) have? Remember to justify your answer. Hint: What does the graph of \( 8x^2 + 31x - 24 \) look like?

9. [16 points] We consider a function \( f(x) \) defined for all real numbers. We suppose that the first and second derivatives \( f'(x) \) and \( f''(x) \) are also defined for all real numbers. Below we show the graph of the second derivative of \( f \). You may assume that \( f''(x) \) is decreasing outside of the region shown.

\[
\begin{array}{c}
\text{y = f''(x)} \\
\end{array}
\]

a. [3 points] Find or estimate the \( x \)-coordinates of all inflection points of \( f(x) \). If there are none, write NONE.

b. [3 points] Find or estimate the \( x \)-coordinates of all inflection points of \( f'(x) \). If there are none, write NONE.

c. [1 point] Suppose that \( f'(0) = 5 \). How many critical points does \( f \) have? For parts d.-f. below, suppose that \( f'(1) = 6.8 \) and \( f(1) = 4 \).

d. [4 points] Let \( Q(x) \) be the quadratic approximation of \( f(x) \) near \( x = 1 \). Find a formula for \( Q(x) \).

e. [2 points] Is the linear approximation of \( f(x) \) near \( x = 1 \) an overestimate or an underestimate of \( f(x) \) for values of \( x \) near 1? Explain your reasoning.

f. [3 points] Let \( L(x) \) be the linear approximation of \( f'(x) \) (the derivative of \( f \)) near \( x = 1 \). Find