8. [11 points] Suppose $J(x)$ is a continuous function defined for all real numbers x. The **derivative** and **second derivative** of $J(x)$ are given by

$$J'(x) = \frac{x^2(x-1)}{\sqrt[3]{x+4}} \qquad \text{and} \qquad J''(x) = \frac{x(8x^2+31x-24)}{3\left(\sqrt[3]{x+4}\right)^4}.$$

a. [2 points] Find the x-coordinates of all critical points of $J(x)$. If there are none, write NONE.

Throughout parts **b.** and **c.** below, you must use calculus to find and justify your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

b. [5 points] Find the x-coordinates of

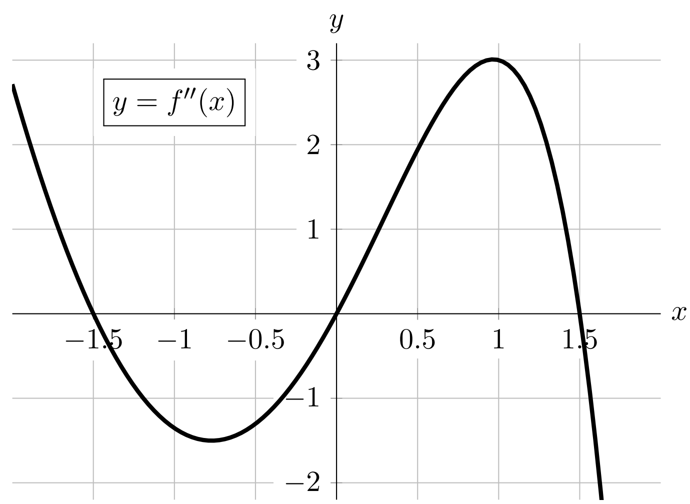
i. all local minima of $J(x)$ and

ii. all local maxima of $J(x)$.

If there are none of a particular type, write NONE.

c. [4 points] The polynomial $8x^2 + 31x - 24$ (from the numerator of $J''(x)$) has two zeroes a and b, where $a \approx -4.54$ and $b \approx 0.66$. How many inflection points does the function $J(x)$ have? Remember to justify your answer. *Hint: What does the graph of* $8x^2 + 31x - 24$ *look like?*

9. [16 points] We consider a function $f(x)$ defined for all real numbers. We suppose that the first and second derivatives $f'(x)$ and $f''(x)$ are also defined for all real numbers. Below we show the graph of the **second derivative** of f. You may assume that $f''(x)$ is decreasing outside of the region shown.



a. [3 points] Find or estimate the x-coordinates of all inflection points of $f(x)$. If there are none, write NONE.

b. [3 points] Find or estimate the x-coordinates of all inflection points of $f'(x)$. If there are none, write NONE.

c. [1 point] Suppose that $f'(0) = 5$. How many critical points does f have?

For parts **d.-f.** below, suppose that $f'(1) = 6.8$ and $f(1) = 4$.

d. [4 points] Let $Q(x)$ be the quadratic approximation of $f(x)$ near $x = 1$. Find a formula for $Q(x)$.

e. [2 points] Is the linear approximation of $f(x)$ near $x = 1$ an overestimate or an underestimate of $f(x)$ for values of x near 1? Explain your reasoning.

f. [3 points] Let $L(x)$ be the linear approximation of $f'(x)$ (the derivative of f) near $x = 1$. Find a formula for $L(x)$.

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