10. [10 points] Consider a continuous function $f(x)$, and suppose that $f(x)$ and its first derivative $f'(x)$ are differentiable everywhere. Suppose we know the following information about $f(x)$ and its first and second derivatives.

- On the interval $(-\infty, -2)$, we have $f(x) = 2^{-x}$.
- $\lim_{x\to\infty} f(x) = 6$.
- $f(2) = -5$, $f(3) = 7$, and $f(4) = 8$.
- $f'(x)$ is equal to 0 at $x = -1$, 2, 4, and not at any other x-values.
- $f''(x) < 0$ on the intervals $-1 < x < 0$ and $3 < x < 5$, and not on any other interval.

For each part below, you must use calculus to find and justify your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

a. [5 points] Find the x-coordinates of

i. the global minimum(s) of $f(x)$ on $[3, \infty)$ and

ii. the global maximum(s) of $f(x)$ on $[3, \infty)$.

If there are none of a particular type, write NONE. If there is not enough information to find a desired x-coordinate, write NEI.

Solution: The only critical point of $f(x)$ on this interval is at $x = 4$.

We have $f(3) = 7$, $f(4) = 8$, and $\lim_{x\to\infty} f(x) = 6$.

We conclude that $\boxed{\text{the global maximum occurs at } x = 4.}$

Since $\lim_{x\to\infty} f(x) < f(3)$, we conclude that $\boxed{\text{there is no global minimum.}}$

b. [5 points] Find the x-coordinates of

i. the global minimum(s) of $f(x)$ on $(-\infty, \infty)$ and

ii. the global maximum(s) of $f(x)$ on $(-\infty, \infty)$.

If there are none of a particular type, write NONE. If there is not enough information to find a desired x-coordinate, write NEI.

Solution: Since 2^{-x} diverges to ∞ as $x \to -\infty$, we conclude that $\boxed{\text{there is no global maximum.}}$

We now must find the global minimum. We know that $f(2) = -5$ and $\lim_{x\to\infty} f(x) = 6$, so there must be a global minimum somewhere. It can only occur at a critical point of $f(x)$, and since $f(4) = 8$, it does not occur at $x = 4$. We now must decide whether the global minimum occurs at $x = 2$ or $x = -1$.

Since $f''(x) < 0$ on the interval $-1 < x < 0$, we see that $f'(x)$ is decreasing on this interval. Since $f'(-1) = 0$, that means that $f'(x)$ must be negative on this interval. That means that $f(x)$ is decreasing on this interval. That means that $f(-0.5) < f(-1)$, and so the function $f(x)$ cannot have a global minimum at $x = -1$.
We conclude that $\boxed{\text{the global minimum occurs at } x = 2.}$