1. [3 points] **There <u>is</u> work to submit for this problem. Read it carefully.**

- You may use your one pre-written page of notes, on an 8.5" by 11" standard sheet of paper, with whatever you want handwritten (not typed) on both sides.
- You are <u>not</u> allowed to use any other resources, including calculators, other notes, or the book.
- You may <u>not</u> use any electronic device or the internet, except to access the Zoom meeting for the exam, to access the exam file itself, to submit your work, or to report technological problems via the Google forms we will provide to do so. The one exception is that you may use headphones (e.g. for white noise) if you prefer, though please note that you need to be able to hear when the end of the exam is called in the Zoom meeting.
- You may <u>not</u> use help from any other individuals (other students, tutors, online help forums, etc.), and may <u>not</u> communicate with any other person about the exam until **10am on Wednesday, March 31** (Ann Arbor time).
- The one exception to the above communication policy is that you may contact the proctors in your exam room via the chat in Zoom if needed.
- Violation of any of the policies above may result in a score of zero for the exam, and, depending on the violation, may result in a failing grade in the course.

As your submission for this problem, you must write "I agree," and write your initials and UMID number to signify that you understand and agree to this policy. By doing this you are attesting that you have not violated and will not violate this policy.

2. [12 points] A table of values for a differentiable, invertible function $g(x)$ and its derivative $g'(x)$ are shown below.

x	0	1	2	3	4	5
$g(x)$	0	0.5	1	2	5	6
$g'(x)$	1.9	1.5	2.8	2.5	2.6	3

a. [2 points] Use the table provided to give the best possible estimate of $g''(3.5)$.

Solution: Since $g''(x)$ is the first derivative of $g'(x)$, we may find the best possible estimate of $g''(3.5)$ by calculating the average rate of change of $g'(x)$ over the smallest available interval that contains 3.5. This interval is $[3, 4]$, and so we compute

$$g''(3.5) \approx \frac{g'(4) - g'(3)}{4-3} = \frac{2.6 - 2.5}{1} = \boxed{0.1.}$$

For parts **b.** and **c.** below, find the **exact** value. Write DNE if the value does not exist, and write NEI if the quantity exists but there is not enough information provided to compute its value.
Your answers should not include the letters g or h but you do not need to simplify. Show work.

b. [3 points] Let $f(x) = g^{-1}(3x)$. Find $f'(2)$.

Solution: Recall the equation $\frac{d}{dt}\left(g^{-1}(t)\right) = \frac{1}{g'(g^{-1}(t))}$. We use this equation along with the Chain Rule to calculate

$$f'(x) = \frac{1}{g'(g^{-1}(3x))} \cdot 3.$$

Plugging in $x = 2$ gives us

$$f'(2) = \frac{3}{g'(g^{-1}(6))} = \frac{3}{g'(5)} = \frac{3}{3} = \boxed{1.}$$

c. [3 points] Let $k(x) = \dfrac{g(x) - 7}{\ln(x)}$. Find $k'(3)$.

Solution: Recall the equation $\frac{d}{dx}\ln(x) = \frac{1}{x}$. We use this equation along with the Quotient Rule to calculate

$$k'(x) = \frac{\ln(x)g'(x) - (g(x) - 7) \cdot \frac{1}{x}}{(\ln(x))^2}.$$

Plugging in $x = 3$ gives us

$$\begin{aligned} k'(3) &= \frac{\ln(3)g'(3) - (g(3) - 7) \cdot \frac{1}{3}}{(\ln(3))^2} \\ &= \frac{\ln(3) \cdot 2.5 - (2 - 7) \cdot \frac{1}{3}}{(\ln(3))^2} \\ &= \boxed{\frac{\ln(3) \cdot 2.5 - \frac{5}{3}}{(\ln(3))^2}.} \end{aligned}$$

Suppose now that $g(x)$ is the number of thousands of seagulls on a beach when there are x hundred tourists on the beach.

d. [4 points] Complete the following sentence to give a practical interpretation of $(g^{-1})'(1.6) = 0.2$.

If the number of seagulls on the beach increases from 1600 *to* 1605 ...

Solution: The function $g^{-1}(s)$ tells us the number of hundreds of tourists on the beach when there are s thousand seagulls. The equation $(g^{-1})'(1.6) = 0.2$ tells us that when there are 1600 seagulls on the beach, then the instantaneous rate of change in the number of tourists is 20 tourists per 1000 seagulls. That is to say, 0.02 tourists per seagull. We therefore may complete the sentence:

If the number of seagulls on the beach increases from 1600 *to* 1605 *the number of tourists increases by about one tenth of a tourist.*

Note: It would also be reasonable to assert that the number of tourists would not change since the change cannot actually be one tenth of a tourist.