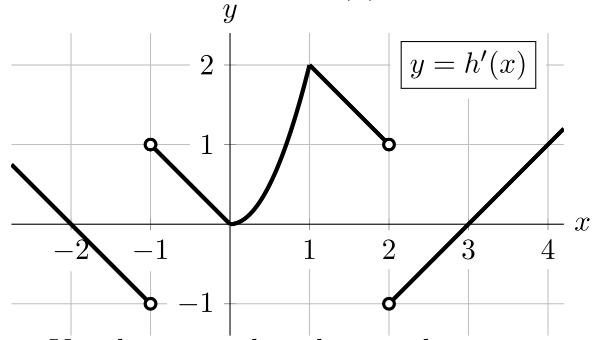
3. [14 points] A table of values for a differentiable, invertible function $g(x)$ and its derivative $g'(x)$ are shown below to the left. (This is the same table as in the previous problem.) Below to the right is shown a portion of the graph of $h'(x)$, the **derivative** of a function $h(x)$. The function $h(x)$ is defined and continuous for all real numbers.

x	0	1	2	3	4	5
$g(x)$	0	0.5	1	2	5	6
$g'(x)$	1.9	1.5	2.8	2.5	2.6	3



Answer parts **a.–c.**, or write NONE if appropriate. You do not need to show work.

a. [2 points] List the x-coordinates of all critical points of $h(x)$ on the interval $(-2, 4)$.

Solution: The critical points of $h(x)$ occur where $h'(x)$ either does not exist or is equal to 0. By looking at the graph, we see that this happens when $\boxed{x = -1, 0, 2, \text{ and } 3.}$

b. [2 points] List the x-coordinates of all critical points of $h'(x)$ on the interval $(-2, 4)$.

Solution: Recall that critical points of $h'(x)$ have to be points *in the domain* of $h'(x)$ where the derivative of $h'(x)$ either is 0 or does not exist. We see no places where the derivative of $h'(x)$ is 0, but we do see that the derivative of $h'(x)$ does not exist when $\boxed{x = 0 \text{ and } x = 1.}$

c. [2 points] List the x-coordinates of all local minima of $h(x)$ on the interval $(-2, 4)$.

Solution: We use the first derivative test, and see that $h'(x)$ is negative to the left of $x = -1$, and is positive to the right of $x = -1$. It has this same sort of behavior for $x = 3$. We conclude that $h'(x)$ has local minima at $\boxed{x = -1 \text{ and } x = 3.}$

d. [8 points] A curve is described implictly by the equation

$$yg(x) = e^{h(x)}.$$

Assume $h(1) = 0$. Then the point $(1, 2)$ lies on this curve.

i. Find $\frac{dy}{dx}$ at the point $(1, 2)$. You must show every step of your work.

Solution: We first find an expression for $\frac{dy}{dx}$. We differentiate the left-hand side of our equation using the Product Rule, and we differentiate the right-hand side using the Chain Rule:

$$\frac{d}{dx}\left(yg(x)\right) = \frac{d}{dx}\left(e^{h(x)}\right)$$
$$yg'(x) + \frac{dy}{dx}g(x) = e^{h(x)}h'(x).$$

We plug in $x = 1$ and $y = 2$ to obtain

$$2g'(1) + \frac{dy}{dx}g(1) = e^{h(1)}h'(1)$$
$$2 \cdot 1.5 + \frac{dy}{dx} \cdot 0.5 = e^0 \cdot 2$$
$$3 + \frac{1}{2}\frac{dy}{dx} = 2$$
$$\frac{dy}{dx} = (2 - 3) \cdot 2 = \boxed{-2}.$$

ii. Write an equation for the tangent line to the curve at the point $(1, 2)$.

Solution: We use point-slope form:

$$y - 2 = -2(x - 1) \quad \text{so} \quad y = 2 - 2(x - 1) \quad \text{or} \quad y = 4 - 2x.$$