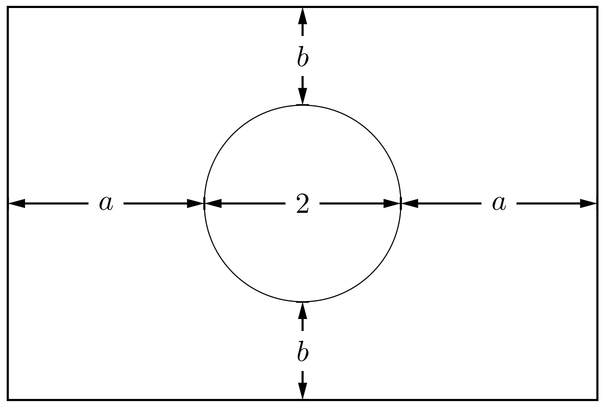
4. [10 points] A landscaper is designing a rectangular garden surrounding a circular fountain in the middle.

- The diameter of the fountain is 2 meters.
- The distance from the fountain to the eastern and western edges of the garden is a meters.
- The distance from the fountain to the northern and southern edges of the garden is b meters.
- The part of the garden **outside of the circular fountain** will be covered with exactly 300 square meters of grass.



a. [4 points] Write a formula for b in terms of a.

Solution: Observe that the radius of the fountain is 1 meter, so that its area is $\pi \cdot 1^2 = \pi$. The length of the rectangle is $2 + 2a$, and the width is $2 + 2b$, so the area of the rectangle is $(2 + 2a)(2 + 2b)$. The area of the part of the garden outside the circular fountain is the difference between these two numbers we have found: $(2 + 2a)(2 + 2b) - \pi$. We are told that this area is equal to 300 meters, and so we have an equation that we can use to solve for b:

$$(2+2a)(2+2b) - \pi = 300$$

$$2 + 2b = \frac{300+\pi}{2+2a}$$

$$b = \frac{1}{2}\left(\frac{300+\pi}{2+2a} - 2\right) = \frac{300+\pi-4a-4}{4a-4}.$$

b. [2 points] Write a formula for the function $P(a)$ which gives the rectangular perimeter of the garden in terms of a only.

Solution: The rectangular perimeter of the garden is equal to $2(2a + 2) + 2(2b + 2)$. We may use our equation $2 + 2b = \frac{300+\pi}{2+2a}$ to write this in terms of a:

$$P(a) = 2(2a+2) + 2\left(\frac{300+\pi}{2+2a}\right).$$

c. [4 points] In the context of this problem, what is the domain of $P(a)$?

Solution: The domain of $P(a)$ is constrained by the following two facts:

- a cannot be less than 0.
- b cannot be less than 0.

From the first fact, we see that the domain of $P(a)$ must begin with "[0." Let us express the second fact as a statement about a. Using the formula $b = \frac{1}{2}\left(\frac{300+\pi}{2+2a} - 2\right)$, we see that the second fact can be expressed as:

$$\begin{aligned} 0 &\leq \frac{1}{2}\left(\frac{300+\pi}{2+2a} - 2\right) \\ 2 &\leq \frac{300+\pi}{2+2a} \\ 4+4a &\leq 300+\pi \\ a &\leq \frac{300+\pi-4}{4}. \end{aligned}$$

Therefore, we see that the domain of $P(a)$ must end with "$\frac{300+\pi-4}{4}$]." Putting these together, we conclude that the domain of $P(a)$ is $\boxed{[0, \frac{300+\pi-4}{4}]}$.