

8. [8 points] Prairie dogs named Paws and Dot have been hard at work digging a tunnel.

Consider the functions L and C defined as follows:

- $L(w)$ is the length of the tunnel, in feet, when w pounds of dirt have been removed.
- $C(w)$ is the total number of Calories the prairie dogs have burned digging their tunnel when they have removed a total of w pounds of dirt for their tunnel.

The functions $L(w)$ and $C(w)$ are both invertible and differentiable.

- a. [4 points] Complete the sentence below to give a practical interpretation of the equation

$$(L^{-1})'(10) = 24.$$

In order to increase the length of the tunnel from 10 feet to 10.25 feet, ...

- b. [4 points]

- i. Which of the following expressions gives the length, in feet, of the prairie dog tunnel when the prairie dogs have burned a total of x Calories digging? Circle the one correct expression.

$$C(L^{-1}(x))$$

$$C^{-1}(L(x))$$

$$L(C^{-1}(x))$$

$$L^{-1}(C(x))$$

- ii. Use the answer you selected in part i to find an expression for the instantaneous rate of change of the length of the prairie dog tunnel, in feet per calorie, when the prairie dogs have burned a total of 2000 calories digging.

Simplify as much as possible. Note that your final answer may involve the function names L , L^{-1} , L' , C , C^{-1} , and C' but should not involve the function names $(L^{-1})'$ or $(C^{-1})'$.

Answer: _____