2. [7 points] A table of values for a differentiable and invertible function \( q(x) \) and its derivative \( q'(x) \) are shown below. Note that this is the same function \( q \) as on the previous page. However, you do not need your work or answers from the previous page to do this problem.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q(x) )</td>
<td>14</td>
<td>10</td>
<td>3</td>
<td>2</td>
<td>-5</td>
<td>-6</td>
<td>-15</td>
</tr>
<tr>
<td>( q'(x) )</td>
<td>-10</td>
<td>-12</td>
<td>-4</td>
<td>0</td>
<td>-2</td>
<td>-5</td>
<td>-6</td>
</tr>
</tbody>
</table>

Let \( C \) be the curve defined implicitly by the equation

\[
xy^2 + \sin(2\pi q(x)) = 6e^{y-4} + 10.
\]

a. [1 point] Exactly one of the following points \((x, y)\) lies on the curve \( C \). Circle that one point.

\((-2, 1)\) \(\boxed{(1, 4)}\) \((0, 4)\) \((0, 10)\)

b. [6 points] Find an equation for the tangent line to the curve \( C \) at the point you chose in part a. Make sure to show your work clearly.

\[
\text{Solution: The slope of this tangent line is equal to } \frac{dy}{dx}\bigg|_{(x,y)=(1,4)}.
\]

To compute this, we first take the derivative with respect to \( x \) of both sides of the given equation for \( C \) and solve for \( \frac{dy}{dx} \).

\[
\frac{d}{dx} (xy^2 + \sin(2\pi q(x))) = \frac{d}{dx} (6e^{y-4} + 10)
\]

\[
y^2 + 2xy \frac{dy}{dx} + 2\pi \cos(2\pi q(x))q'(x) = 6e^{y-4} \frac{dy}{dx}
\]

\[
(2xy - 6e^{y-4}) \frac{dy}{dx} = -y^2 - 2\pi \cos(2\pi q(x))q'(x)
\]

\[
\frac{dy}{dx} = \frac{-y^2 - 2\pi \cos(2\pi q(x))q'(x)}{2xy - 6e^{y-4}}
\]

So at the point \((1, 4)\),

\[
\frac{dy}{dx}\bigg|_{(x,y)=(1,4)} = \frac{-y^2 - 2\pi \cos(2\pi q(x))q'(x)}{2xy - 6e^{y-4}}
\]

\[
= \frac{-(4)^2 - 2\pi \cos(2\pi q(1))q'(1)}{2(1)(4) - 6e^{4-4}}
\]

\[
= \frac{-16 - 2\pi \cos(-10\pi)(-2)}{8 - 6}
\]

\[
= \frac{-16 + 4\pi}{2}
\]

\[
= 2\pi - 8.
\]

Therefore, an equation for the tangent line to the curve \( C \) at the point \((1, 4)\) is

\[
y = 4 + (2\pi - 8)(x - 1)
\]

Answer: \( y = 4 + (2\pi - 8)(x - 1) \)