2. [7 points] A table of values for a differentiable and invertible function $q(x)$ and its derivative $q'(x)$ are shown below. Note that this is the same function q as on the previous page. However, you do not need your work or answers from the previous page to do this problem.

x	-3	-2	-1	0	1	2	3
$q(x)$	14	10	3	2	-5	-6	-15
$q'(x)$	-10	-12	-4	0	-2	-5	-6

Let $\mathcal{C}$ be the curve defined implicitly by the equation

$$xy^2 + \sin(2\pi q(x)) = 6e^{y-4} + 10.$$

a. [1 point] Exactly one of the following points (x, y) lies on the curve $\mathcal{C}$. Circle that one point.

$(-2, 1)$ $\boxed{(1, 4)}$ $(0, 4)$ $(0, 10)$

b. [6 points] Find an equation for the tangent line to the curve $\mathcal{C}$ at the point you chose in part **a**. Make sure to show your work clearly.

Solution: The slope of this tangent line is equal to $\left.\frac{dy}{dx}\right|_{(x,y)=(1,4)}$. To compute this, we first take the derivative with respect to x of both sides of the given equation for $\mathcal{C}$ and solve for $\frac{dy}{dx}$.

$$\begin{aligned}
\frac{d}{dx}\left(xy^2 + \sin(2\pi q(x))\right) &= \frac{d}{dx}\left(6e^{y-4} + 10\right) \\
y^2 + 2xy\frac{dy}{dx} + 2\pi\cos(2\pi q(x))q'(x) &= 6e^{y-4}\frac{dy}{dx} \\
(2xy - 6e^{y-4})\frac{dy}{dx} &= -y^2 - 2\pi\cos(2\pi q(x))q'(x) \\
\frac{dy}{dx} &= \frac{-y^2 - 2\pi\cos(2\pi q(x))q'(x)}{2xy - 6e^{y-4}}.
\end{aligned}$$

So at the point $(1, 4)$,

$$\begin{aligned}
\left.\frac{dy}{dx}\right|_{(x,y)=(1,4)} &= \frac{-y^2 - 2\pi\cos(2\pi q(x))q'(x)}{2xy - 6e^{y-4}} \\
&= \frac{-(4)^2 - 2\pi\cos(2\pi q(1))q'(1)}{2(1)(4) - 6e^{4-4}} \\
&= \frac{-16 - 2\pi\cos(-10\pi)(-2)}{8 - 6} \\
&= \frac{-16 + 4\pi}{2} \\
&= 2\pi - 8.
\end{aligned}$$

Therefore, an equation for the tangent line to the curve $\mathcal{C}$ at the point $(1, 4)$ is

$$y = 4 + (2\pi - 8)(x - 1)$$

Answer: $y =$ $4 + (2\pi - 8)(x - 1)$