3. [10 points] Suppose $B(x)$ is a continuous function defined for all real numbers x. The derivative and second derivative of $B(x)$ are given by

$$B'(x) = e^{-x}x(\sqrt[3]{x}-2)^2 \quad \text{and} \quad B''(x) = \frac{-e^{-x}(x-3)(3x-2)}{3\sqrt[3]{x}-2}.$$

a. [5 points] Find the exact x-coordinates of all local minima and local maxima of $B(x)$. If there are none of a particular type, write NONE. You must use calculus to find and justify your answers. Be sure your conclusions are clearly stated and that you show enough evidence to support them.

Solution: We begin by finding the critical points of $B(x)$.
Note that $B'(x) = 0$ when $x = 0$ and $x = 2$. Both of these x-coordinates are in the domain of $B(x)$, so they are both critical points of B.
Since $B'(x)$ is defined for all x, there are no points where $B(x)$ is not differentiable, so the only critical points of $B(x)$ are $x = 0$ and $x = 2$. We next apply the First Derivative Test to classify these critical points.

(checking signs of $B'(x)$)	$x<0$	$0<x<2$	$2<x$
e^{-x}	$+$	$+$	$+$
x	$-$	$+$	$+$
$(\sqrt[3]{x}-2)^2$	$+$	$+$	$+$
$B'(x) = e^{-x}x(\sqrt[3]{x}-2)^2$	$+\cdot-\cdot+=-$	$+\cdot+\cdot+=+$	$+\cdot+\cdot+=+$

This gives the following number line for $B'(x)$:

$B'(x)$	$+\cdot-\cdot+=-$		$+\cdot+\cdot+=+$		$+\cdot+\cdot+=+$
x		0		2	

By the First Derivative Test, since $B(x)$ is continuous, $B(x)$ has a local minimum at $x = 0$ and no local maxima.

Answer: Local min(s) at $x =$ ___0___ and Local max(es) at $x =$ ___**None**___

b. [5 points] Find the exact x-coordinates of all inflection points of $B(x)$, or write NONE if there are none. You must use calculus to find and justify your answers. Be sure your conclusions are clearly stated and that you show enough evidence to support them.

Solution: To find candidate inflection points, we check when $B''(x) = 0$ and $B''(x)$ does not exist. $B''(x) = 0$ when $x = 3$ and $x = \frac{2}{3}$. $B''(x)$ does not exist when $x = 2$. These are all in the domain of $B(x)$ so $x = \frac{2}{3}, 2, 3$ are the candidate inflection points of $B(x)$. We now consider the sign of $B''(x)$ to determine which of these candidates are actually inflection points.

(checking signs of $B''(x)$)	$x<\frac{2}{3}$	$\frac{2}{3}<x<2$	$2<x<3$	$3<x$
$-e^{-x}$	$-$	$-$	$-$	$-$
$(x-3)$	$-$	$-$	$-$	$+$
$(3x-2)$	$-$	$+$	$+$	$+$
$\sqrt[3]{x}-2$	$-$	$-$	$+$	$+$
$B''(x) = \dfrac{-e^{-x}(x-3)(3x-2)}{3\sqrt[3]{x}-2}$	$\frac{-\cdot-\cdot-}{-}=+$	$\frac{-\cdot-\cdot+}{-}=-$	$\frac{-\cdot-\cdot+}{-}=-$	$\frac{-\cdot+\cdot+}{+}=-$

This gives the following number line for $B''(x)$:

$B''(x)$	$\frac{-\cdot-\cdot-}{-}=+$		$\frac{-\cdot-\cdot+}{-}=-$		$\frac{-\cdot-\cdot+}{-}=-$		$\frac{-\cdot+\cdot+}{+}=-$
x		$2/3$		2		3	

Because the sign of the second derivative changes at each of the candidate inflection points, all of $x = \frac{2}{3}, 2, 3$ are indeed inflection points.

Answer: Inflection point(s) at $x =$ ___$\frac{2}{3}, 2, 3$___